

## Optimal sizing and siting distributed generation resources using a multiobjective algorithm

Seyed Amir HOSSEINI,<sup>1</sup> Seyed Siavash Karimi MADAHİ,<sup>2</sup> Farzad RAZAVI,<sup>1,\*</sup>  
Mohsen KARAMI,<sup>3</sup> Ali Asghar GHADIMI<sup>4</sup>

<sup>1</sup>Department of Electrical Engineering, University of Tafresh, Tafresh, Iran

<sup>2</sup>Department of Power Engineering, Faculty of Technical Sciences, University of Novi Sad, Novi Sad, Serbia

<sup>3</sup>Department of Electrical Engineering, Islamic Azad University, Ashtian Branch, Ashtian, Iran

<sup>4</sup>Department of Electrical Engineering, Faculty of Engineering, Arak University, Arak, Iran

Received: 20.08.2011

• Accepted: 15.03.2012

• Published Online: 03.05.2013

• Printed: 27.05.2013

**Abstract:** The restructuring of the electrical market, improvement in the technologies of energy production, and energy crisis have paved the way for increasing applications of distributed generation (DG) resources in recent years. Installing DG units in a distribution network may result in positive impacts, such as voltage profile improvement and loss reduction, and negative impacts, such as an increase in the short-circuit level. These impacts depend on the type, capacity, and place of these resources. Therefore, finding the optimal place and capacity of DG resources is of crucial importance.

Accordingly, this paper is aimed at finding the optimal place and capacity of DG resources, in order to improve the technical parameters of the network, including the power losses, voltage profile, and short-circuit level. The proposed formulation of this paper significantly increases the convergence and the speed of the finding the answers. Furthermore, to select the optimal weighting coefficients, an algorithm is proposed. The weighting coefficients are decided on according to the requirements of each network and deciding on them optimally prevents the arbitrarily selection of these resources. The genetic algorithm is used to minimize the objective function and to find the best answers during the investigation. Finally, the proposed algorithm is tested on the Zanzan Province distribution network in Iran and the simulation results are presented and discussed.

**Key words:** DG optimal location, DG optimal size, distributed generation, objective function, genetic algorithm

### 1. Introduction

Distributed generation (DG) is defined as electrical power resources that are directly connected to the network [1]. These resources include renewable and nonrenewable energies. Renewable energies that are applicable for DG include wind, solar, and biomass. On the other hand, nonrenewable energies include microturbines, gas turbines, and fuel cells [2,3]. Installing DG on distribution networks has many different impacts on the parameters of these networks. These impacts can be positive and negative. The positive impacts of installing DG resources includes increasing the power quality, improving the voltage profile, reducing the power loss, decreasing the requirements of installing new transmission lines, and deferring the necessity of improving the capacity of substations [4,5].

On the other hand, the main adverse impact of installing DG is the increase in the short-circuit level of the network [6].

\*Correspondence: farzad.razavi@tafreshu.ac.ir

DGs are only beneficial if their installations are carried out according to the appropriate plans [7]. Studies show that if the capacity and location of DGs are not identified appropriately, not only are the network parameters not improved, but they are also deteriorated [8,9]. Thereby, 2 of the most important factors of DG plans are identifying the capacity and the location of these resources [2]. The place and capacity of DGs can be decided according to the improvement of one or more parameters in order to increase the efficiency and decrease the adverse effects of installing them. However, siting and sizing DGs with the aim of improving a single parameter enhances the considered parameter significantly, but may have a negative impact on the other parameters of the network. On the other hand, siting and sizing DGs with the purpose of enhancing some of the parameters of the network will result in the improvement of the considered parameters. Considering the impacts of different parameters is an important issue, while having a multiobjective siting and sizing. These impacts are identified using the weighting coefficients for different parameters [1]. The values of these coefficients are generally identified arbitrarily and according to the point of view of the designer. Hence, in most cases, some benefits of these resources are not acquired. Many studies have been carried out about the issue of siting and sizing, which can be broadly classified as technical and commercial studies.

Technical studies generally consider the power loss, short-circuit level, voltage profile, and power quality issues [6]. In [1], the optimal place and capacity of DG was decided in order to improve the voltage profile, reduce power loss, and diminish the short-circuit level of the buses. The proposed formulation of this paper was based on a heuristic method. The authors in [3] proposed an analytical method to identify the location and capacity of a single DG. The main objective of this work was diminishing the power loss and deciding the appropriate power factor of the DG. The work in [4] considered the reduction of the power loss and satisfaction of the voltage and power constraints as the main objectives of siting and sizing. The proposed formulation of this work was applicable in unbalanced distribution networks. The major defect of this paper was deciding the place and capacity of only a single DG. In [5], the place and capacity were found according to the reduction of the power loss. An analytical method was used in this paper that does not require calculating the admittance and impedance matrices. The voltage profile was the only considered constraint of this paper. The objective of the siting and sizing in [6] was satisfying the different constraints and reaching the maximum penetration level of the DG. The considered constraints of this paper included the flowing power of the terminals, the capacity of transformers, voltage profile, and short-circuit level. In [8], the optimal capacity and location of the DG was decided based on an exact formulation and the use of consecutive load-flows and the power loss sensitivity. In this method, calculating the  $Z_{bus}$  and  $Y_{bus}$  matrices was not required. However, this method was not applicable in real networks due to the size, complexity, and special conditions of real distribution networks. The authors in [10] proposed a formulation in order to decide the optimal place and capacity of the DG to reduce the power loss in 2 situations. In the 1st situation, the DG units supplied the required power of the loads (unidirectional) and in the 2nd situation not only did the DG units supply the required power of the loads, but also they also injected some power into the upper network (bidirectional). Injecting power into an upper network is usually prevented by the electrical companies due to the protective problems that it causes. In [11], an analytical method was proposed to decide the optimal place and capacity of the DG units in radial networks. This paper was aimed at minimizing the power loss. The proposed method of this work was based on using the  $Z_{bus}$  and  $Y_{bus}$  matrices.

Commercial papers are generally associated with DG costs, installation costs, operation costs, expected benefits, and the benefits of power loss reduction [6]. The authors in [12] proposed a 2-stage method in order to minimize the costs and increase the benefits of DG installation while sizing and siting. In the 1st stage, the

costs were minimized and in the 2nd stage the benefits of installing DG were maximized. Ahmadigorji et al. [13] decided the optimal place and capacity of DG by maximizing the benefit of DG and minimizing the costs of it. In addition, the authors have sited and sized DG units in future networks, considering the cost of substituting the resources. The place and capacity of DG can also be found using nodal pricing [14]. The subject of this paper was obtaining the most possible benefit resulting from the power loss reduction and the placement of the voltage profile in an allowable range. The nodal prices included the cost of the active and reactive power in the location of the buses and the cost of the power loss. The authors in [15] used a value-based method to balance the cost and benefit of DG. This work has identified the best type of DG, aside from siting and sizing it. Using the proposed method in this paper, the benefits of increasing the power quality, decreasing the customer's outages, and decreasing the cost of energy savings are expected. The problem of sizing and siting of DG units on distribution feeders was simulated and solved based on multipurpose optimization in [16]. The reduction of the constant and variable costs were considered in the model objectives. In [17], a repetitive search algorithm on MV feeders to decide the optimal location and capacity of DG was used. Minimum investment and operation costs such as power loss and supplying the required power for the network were considered as objectives. The authors in [18] and [19] presented some indices, such as the power loss index, voltage profile index, and short-circuit index, in order to evaluate the efficiency of installed DG units.

This paper proposes a method for simultaneously finding the optimal location and size of a number of DG resources in order to improve the technical parameters of the network, such as power loss, the voltage profile, and short-circuit level of the buses. Some constraints are also considered during the procedure of sizing and siting. Since most of the electrical power companies in Iran are interested in using the DG resources with capacities of 5, 10, and 15 MW, this paper considers these amounts as the possible capacities of the DG resources. The proposed method is also capable of deciding the optimal number of required DG units, based on the existent constraints and providing the most possible benefit of installing them. In addition, this paper proposes a method to decide the weighting coefficients with high accuracy, according to the requirements of each network. Using this method to select the weighting coefficients, the most possible benefits of installing DG resources are acquired. Finally, the proposed method is tested on an actual power network in Zanzan Province, in Iran, and the simulation results are presented and discussed.

The structure of the paper is as follows. The problem is staged and some defects of the previous works are presented in Section 2. Section 3 presents the new method and the proposed formulation. The optimization algorithm, sample network, software, and the method of applying the proposed method to the network are presented in Section 4, and the simulation results are presented and discussed in Section 5, and finally, in Section 6, the method for correcting the weighting coefficients is presented.

## 2. Problem statement

One of the major defects in previous works of siting and sizing is considering only one objective for the problem. In these studies, the impact of installing DG resources on the other parameters of the network is not considered. To clarify the problem, 2 radial networks, with and without DG are considered, as illustrated in Figures 1 and 2. The considered load of the network is a balanced 3-phase load with the constant power of  $S_L$ , which is located at the end of the feeder.

### 2.1. The impact of DG on power loss

Since installing DG resources brings about the reduction of the flowing current of the lines, they can reduce the amount of power losses [19].

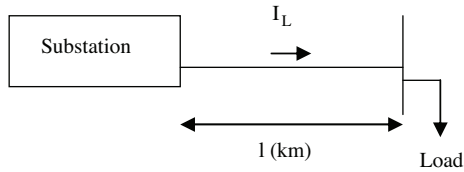


Figure 1. The considered network without DG.

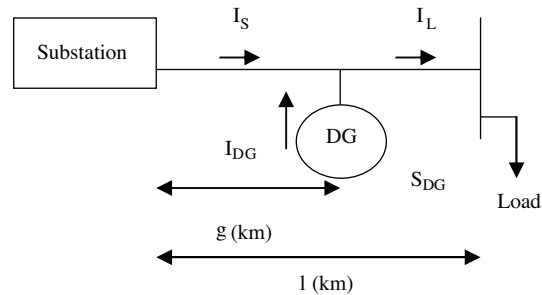


Figure 2. The considered network with DG.

The amount of power loss in the absence of DG can be calculated by the following equation:

$$I_S = I_L \tag{1}$$

$$P_{loss}^{Without DG} = 3.r.l. |I_L|^2 \tag{2}$$

where  $P_{Loss}^{Without DG}$  is the power loss in the absence of DG,  $r$  is the resistance of the line per length, and  $I_L$  is the current of customer terminal.

The amount of power loss in the presence of DG can be calculated by the following equation:

$$P_{loss}^{With DG} = P_{loss1} + P_{loss2} \tag{3}$$

where  $P_{Loss}^{With DG}$  is the power loss in the presence of DG,  $P_{Loss1}$  is the power loss from the substation to DG location, and  $P_{Loss2}$  is the power loss from DG location to the load location.

$$P_{loss}^{With DG} = 3rg |I_S|^2 + 3(l - g)r |I_L|^2 \tag{4}$$

Therefore, the variation of the power loss after installing DG is obtained using Eq. (5).

$$\Delta P_{loss} = 3rg |I_S|^2 - 3rg |I_L|^2 \tag{5}$$

Eq. (6) stands in 3-phase systems.

$$I = \left(\frac{S}{3 \cdot V}\right)^* \tag{6}$$

After installing DG, the following equation stands:

$$|I_S| = |I_L - I_{DG}| \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (5), the following equation is obtained:

$$\Delta P_{loss} = \frac{rg \cdot (|S_L^* \cdot V_{DG}^* - S_{DG}^* \cdot V_L^*|^2 - |S_L^* \cdot V_{DG}^*|^2)}{3(|V_L^*| \cdot |V_{DG}^*|)^2} \tag{8}$$

It can be concluded from Eq. (8) that identifying an appropriate capacity for a DG unit plays a pivotal role in reducing the power loss. On the other hand, if the considered capacity is inappropriate, it may increase the power loss.

**2.2. The impact of DG on voltage profile**

Installing DG on distribution feeders may result in the improvement or deterioration of the voltage profile of the customer terminal. The voltage profile of the customer in the absence of DG is calculated using Eq. (9).

$$V_L^{Without DG} = V_S - (r + jx) \cdot l \cdot I_L \tag{9}$$

where  $V_S$  is the voltage of the substation and is assumed to be 1 pu. The voltage of the customer in the existence of DG is calculated using Eq. (10).

$$V_L^{With DG} = V_S - (r + jx) \cdot g \cdot I_S - (r + jx) \cdot (l - g) \cdot I_L \tag{10}$$

Comparing Eqs. (9) and (10), the variation in the voltage of the customer after installing DG is calculated.

$$\Delta V_L = (r + jx) \cdot g \cdot (I_L - I_S) = (r + jx) \cdot g \cdot I_{DG} \tag{11}$$

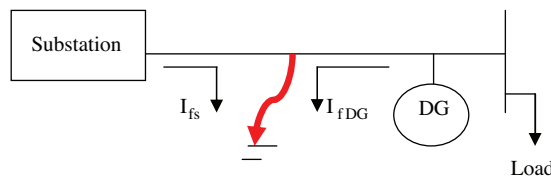
Using Eqs. (6) and (7), the amount of increase in the voltage of the customer after installing DG is obtained.

$$|\Delta V_L| = \frac{\sqrt{r^2 + x^2} \cdot g \cdot |S_{DG}|}{3 |V_{DG}|} \tag{12}$$

It is apparent from Eq. (12) that ignoring the amount of produced power of DG may result in too much of an increase in the voltage profile, and thereby it may bring about exceeding the allowable limit.

**2.3. The impact of DG on the short-circuit level**

A DG unit is almost always placed in parallel with the network. Hence, the calculated impedance from a fault point of view diminishes and the fault current level increases. Increasing the DG unit capacity, the equivalent impedance of the DG unit diminishes and the short-circuit level of the network changes, as illustrated in Figure 3.



**Figure 3.** Increase in the short-circuit level in the presence of DG.

The short-circuit level in the absence of DG is equal to:

$$|I_{F_{out}}| = |I_{fs}| \tag{13}$$

Therefore, the fault current is equal to the flowing current of the substation before installing DG and its value changes into Eq. (14) after installing DG.

$$|I_{F_{out}}| = |I_{fs} + I_{fDG}| \tag{14}$$

If after installing the DG resources the increase in the short-circuit level exceeds the tolerable range of the circuit-breakers (CBs), they must be substituted with suitable CBs. This imposes an excessive cost on electrical

companies. Therefore, the short-circuit level of the buses should be considered while siting and sizing the DG resources.

Another defect of previous works is associated with deciding the weighting coefficients. Having a significant impact on the acquired answers, weighting coefficients are of great importance in acquiring the maximum efficiency of DG resources. Thereby, having an accurate method to identify the weighting coefficients seems to be essential.

### 3. New method

This paper is aimed at proposing an algorithm to find the best place, capacity, and number of DG resources according to the technical parameters in the distribution network. Therefore, an objective function is proposed that includes the most important parameters of the network. The considered parameters include the total power loss of the network, voltage profile of the distribution buses, short-circuit level of the distribution buses, and the appropriate number of DG units. In this section, the structure of the objective function (OF) is presented, a method is presented to identify the weighting coefficients of each parameter, the considered constraints of sizing and siting are presented, and the indices are introduced to evaluate the efficiency of the proposed method.

#### 3.1. The objective function

To minimize a function consisting of some parameters, the general function can be written as a summation of those parameters.

$$f = f_1 + f_2 + \dots + f_N = \sum_{i=1}^N f_i \quad (15)$$

Here, N is the number of factors that affect the OF. It is assumed to be 4 in this paper. Each of these factors is presented in the following.

##### 3.1.1. Parameter of the ‘total power loss of the network’

The power loss of the network is calculated in Eq. (16).

$$f_1 = f(P_{loss}) = P_{loss} \quad (16)$$

Here,  $P_{loss}$  is the total power loss of the network. Normalizing the  $P_{loss}$  and considering  $\alpha$  for a weighting coefficient, the final function of  $f_1$  is acquired from Eq. (17).

$$f_1 = \alpha \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} \quad (17)$$

##### 3.1.2. Parameter of the ‘voltage profile’

The voltage profile parameter depends on the voltage of the buses. This parameter is defined as a variation of the voltage from the ideal value of 1 pu.

$$v = V_{bus}^{withDG} - 1(Pu) \quad (18)$$

Since the function of  $\nu$  is a pu value, it is not necessary to normalize it. To identify the voltage variation of all of the buses from 1 pu, the V vector is defined as in Eq. (19):

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \tag{19}$$

where n is the number of distribution network buses:

$$v_k = V_{bus,k}^{withDG} - 1, \quad k = 1, 2, \dots, n \tag{20}$$

where  $v_k$  is the voltage variation of the  $k$ th bus from 1 pu. The function of  $f_2$  is acquired, calculating the voltage variation of each bus from 1 pu:

$$f_2(V_{bus}) = V^T B V \tag{21}$$

where B is the matrix of the weighting coefficients and is defined as in Eq. (22).

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}_{n \times n} \tag{22}$$

Therefore,  $f_2$  equals:

$$f_2(V_{bus}) = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}^T \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \tag{23}$$

By simplifying Eq. (23), Eq. (24) is obtained as follows:

$$f_2(V_{bus}) = \sum_{i=1}^n \sum_{j=1}^n b_{ij} v_i v_j \tag{24}$$

Replacing Eq. (20) in Eq. (24) results in Eq. (25).

$$f_2(V_{bus}) = \sum_{k=1}^n b_{kk} (V_{bus,k}^{withDG} - 1)^2 + \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n b_{ij} (V_{bus,i}^{withDG} - 1)(V_{bus,j}^{withDG} - 1) \tag{25}$$

The voltages of 2 different buses have no impact on each other; thus the value of  $b_{ij}$  for  $i \neq j$  is assumed to be 0. The voltage profile, thereby, equals the following equation:

$$f_2(V_{bus}) = \sum_{k=1}^n b_k (V_{bus,k}^{withDG} - 1)^2 \tag{26}$$

**3.1.3. The parameter of the ‘short-circuit level’**

The function of  $w$  is defined as in Eq. (27). This function shows the difference in the short-circuit current in the presence and absence of DG.

$$w = i_{sc}^{withDG} - i_{sc}^{withoutDG} \tag{27}$$

Normalizing  $w$  results in Eq. (28).

$$w = \frac{i_{sc}^{withDG} - i_{sc}^{withoutDG}}{i_{sc}^{withDG}} \tag{28}$$

The  $W$  vector is defined as in Eq. (29) to show the variation in the short-circuit level of all of the buses in the absence and existence of DG.

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{29}$$

Each element of  $W$  is calculated by Eq. (30).

$$w_k = \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}}, \quad k = 1, 2, \dots, n \tag{30}$$

According to the foregoing equations, the function of the short-circuit level is calculated as follows:

$$f_3(i_{sc}) = W^T C W \tag{31}$$

where  $C$  is the matrix of the weighting coefficients and is defined as follows:

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}_{n \times n} \tag{32}$$

$f_3$  is calculated in Eq. (33) with respect to Eq. (32).

$$f_3(i_{sc}) = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}^T \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{33}$$

Simplifying the foregoing equation, the parameter of the short-circuit level is obtained in Eq. (34).

$$f_3(i_{sc}) = \sum_{i=1}^n \sum_{j=1}^n c_{ij} w_i w_j \tag{34}$$



Substituting Eq. (30) into (34) results in the following equation:

$$f_3(i_{sc}) = \sum_{k=1}^n c_{kk} \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n c_{ij} \left( \frac{i_{sc,i}^{withDG} - i_{sc,i}^{withoutDG}}{i_{sc,i}^{withDG}} \right) \left( \frac{i_{sc,j}^{withDG} - i_{sc,j}^{withoutDG}}{i_{sc,j}^{withDG}} \right) \quad (35)$$

The differences in the short-circuit levels of 2 different buses have no impact on each other; thus the value of  $c_{ij}$  for  $i \neq j$  is assumed to be 0. The parameter of the short-circuit level, therefore, equals the following equation:

$$f_3(i_{sc}) = \sum_{k=1}^n c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 \quad (36)$$

### 3.1.4. The parameter of ‘capacity of DG units’

Installing and exploiting some DG resources with a small capacity is proven to be more effective than with a single DG with a big capacity [9,14]. Therefore, one of the objectives of this paper is sizing and siting a number of DG units with a small capacity. To achieve this purpose,  $f_4$  is defined as follows:

$$f_4(CG) = \sum_{k=1}^n d_k CG_k \quad (37)$$

where  $CG_K$  is the capacity of the installed DG on the  $k$ th bus and in MVA, and  $d_K$  is the related weighting coefficient. Dividing Eq. (37) into  $S_{base}$  normalizes the  $f_4$ .  $S_{base}$  is the base value of the actual power of the network.

$$f_4(CG) = \sum_{k=1}^n d_k \frac{CG_k}{S_{base}} \quad (38)$$

### 3.1.5. Summarize

According to the preceding equations, the final OF to be minimized is acquired as follows:

$$f = f_1 + f_2 + f_3 + f_4 \quad (39)$$

Substituting  $f_1, f_2, f_3,$  and  $f_4$  by their obtained values will result in the following equation:

$$f = a \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + \sum_{k=1}^n b_k (V_{bus,k}^{withDG} - 1)^2 + \sum_{k=1}^n c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + \sum_{k=1}^n d_k \frac{CG_k}{S_{base}} \quad (40)$$

By simplifying Eq. (40), Eq. (41) is obtained as follows:

$$f = \sum_{k=1}^n \left( a_k \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + b_k (V_{bus,k}^{withDG} - 1)^2 + c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + d_k \frac{CG_k}{S_{base}} \right) \quad (41)$$

Since all of the buses are equally as important, the weighting coefficients of all of the buses are assumed to be equal. Therefore,

$$a_1 = a_2 = \dots = a_n = a, \quad b_1 = b_2 = \dots = b_n = b, \quad c_1 = c_2 = \dots = c_n = c, \quad d_1 = d_2 = \dots = d_n = d \quad (42)$$

Consequently, the considered OF is as follows:

$$f = \sum_{k=1}^n \left( a \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + b(V_{bus,k}^{withDG} - 1)^2 + c \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + d \frac{CG_k}{S_{base}} \right) \quad (43)$$

### 3.2. Constraints

Constraints are an issue of great importance in optimization procedures. Indeed, an optimal answer is an answer that satisfies all of the constraints of the optimization problem. In this paper, the following technical constraints are considered while siting and sizing.

#### 3.2.1. Constraint of the ‘voltage of the buses’

The variation range of all of the distribution buses should be within a specified limit [12,20].

$$V_{min} < V_{bus,k}^{WithDG} < V_{max} \quad (44)$$

Here,  $V_{min}$  (0.95 (pu)) and  $V_{max}$  (1.05 (pu)) are the limits of the allowable voltages. Since the values of the voltages were inserted into the OF as a square of their variations from 1 pu, the following equation stands:

$$0.95 < V_{bus,k}^{withDG} < 1.05 \rightarrow |V_{bus,k}^{withDG} - 1| < 0.05 \rightarrow (V_{bus,k}^{withDG} - 1)^2 < 0.0025 \quad (45)$$

#### 3.2.2. Constraint of the ‘short-circuit current in the existence of DG units’

The increase in the short-circuit level, in the presence of DG, must be within the tolerable range of the CBs. Thus, Eq. (46) should stand [1]:

$$I_{sc,k}^{withDG} < \text{Short circuit level of the currently installed protective devices} \quad (46)$$

#### 3.2.3. The summation of the ‘DG capacities’

The maximum of the summation of the produced active power of the DG units must not exceed the load demand of the network [3].

$$\sum_{k=1}^{NDG} CG_k \leq P_{load} \quad (47)$$

Here,  $N_{DG}$  is the number of DG units. Observing this constraint prevents a bidirectional power flow.

#### 3.2.4. The constraint of ‘the minimum power factor of the DG units’

Synchronous generators are capable of producing active and reactive power, simultaneously [3]. Since electrical companies are more interested in operating in the upper power factors, this constraint should be considered while sizing and siting. The mentioned constraint is defined as follows:

$$0.8 \leq PF_{DG,k} \leq 1, \quad k = 1, 2, \dots, n_{DG} \quad (48)$$

where  $PF_{DG,K}$  is the power factor of each DG unit. In this paper, the power factors are assumed to be 0.8.

### 3.2.5. Constraint of the loading of ‘distribution substations’

Installing DG units should not increase the loading of transformers more than their allowable range [6,10]. Otherwise, the electrical companies have to replace the overloaded transformers.

$$Trans_{Loading} \leq 100\% \tag{49}$$

Constraints can be divided into soft and hard constraints. Hard constraints are those in which violating them is not permitted and soft constraints are those that can be violated to some extent [10]. To prevent any changes in the distribution network as a result of installing DG, all of the considered constraints of this paper are assumed to be hard.

### 3.3. Identifying the weighting coefficients

This paper presents a method to identify the weighting coefficients with a high accuracy in 2 stages. In the first stage, some values are selected for the weighting coefficients of Eq. (43) with respect to the following equations. In the second stage, the network is simulated and the optimal place and capacity are decided, and the conformity of the considered weighting coefficients is analyzed.

#### 3.3.1. The first stage of ‘identifying the weighting coefficients’

As presented in the previous section, the acquired OF has 4 factors. Since the relative importance of all of the factors is equal, the variation of each factor can be taken into the same range, using the weighting coefficients.

##### A. The weighting coefficient of the ‘power loss’

The power losses can be reduced with the appropriate siting and sizing of the DG resources. This value can be increased down to 0, but this is not economical due to the cost of the DG units. It is proven that assuming  $\frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} = 0.5$ , the weighting coefficient of the power loss can have the most economical value. The weighting coefficient of  $\alpha$  is therefore calculated as Eq. (50).

$$a = \frac{1}{0.5} = 2 \tag{50}$$

##### B. The weighting coefficient of the ‘voltage profile’

$b$  stands for the weighting coefficient of the voltage, and with respect to Eq. (43), it can be obtained from the following equation:

$$b = \frac{1}{0.0025} = 400 \tag{51}$$

##### C. The weighting coefficient of the ‘short-circuit level’

The results of different simulations on test networks show that by installing DG the short-circuit level can be increased by up to 150% more than in the previous stage. Namely, the short-circuit level must be multiplied by 2.5. Therefore,

$$i_{sc,k}^{withDG} = 2.5i_{sc,k}^{withoutDG} \rightarrow \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} = 0.6 \rightarrow \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 = 0.36 \tag{52}$$

As a consequence, the weighting coefficient of c equals:

$$c = \frac{1}{0.36} = 2.78 \tag{53}$$

**D. The weighting coefficient of the ‘capacity of DG units’**

The assumed capacities of the DG units are 5, 10, and 15 MW, the power factor is assumed to be 0.8 lag, and  $S_{base}$  is assumed to be 100 MVA in this paper. Therefore, the following equation stands:

$$CG_k = \frac{P_{DG,k}}{\cos \phi} = \frac{10}{0.8} = 12.5MVA \rightarrow \frac{CG_k}{S_{base}} = \frac{12.5}{100} = 0.125 \tag{54}$$

Therefore, the weighting coefficient of d is equal to:

$$d = \frac{1}{0.125} = 8 \tag{55}$$

The obtained weighting factors should be normalized to have the best answers.

$$a = \frac{a}{a+b+c+d}, \quad b = \frac{b}{a+b+c+d}, \quad c = \frac{c}{a+b+c+d}, \quad d = \frac{d}{a+b+c+d} \tag{56}$$

Normalizing the weighting coefficients, their summation turns out to be 1 [1].

$$a + b + c + d = 1 \tag{57}$$

As a result, the initial values of the weighting coefficients are obtained.

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 4.85 \\ 969.04 \\ 6.73 \\ 19.38 \end{bmatrix} \times 10^{-3} \tag{58}$$

**3.4. Indices**

The existence of some indices plays a crucial role in evaluating the efficiency of the sizing and siting on the technical parameters of the network. Hence, some indices are introduced. The introduced indices can illustrate the average of the variation of the parameters. In addition, they can identify whether the parameters are in their allowable range or not.

**3.4.1. The index of ‘the power loss’**

Using this index, the amount of variation of the active and reactive power losses, which is a result of installing the DG resources, can be calculated. This index is defined as follows for the active and reactive power losses [21]:

$$IL_p = \left(1 - \frac{Re \{Losses_{With DG}\}}{Re \{Losses_{Without DG}\}}\right) \times 100\% \tag{59}$$

$$IL_q = \left(1 - \frac{Im \{Losses_{With DG}\}}{Im \{Losses_{Without DG}\}}\right) \times 100\% \tag{60}$$

where  $IL_p$  and  $IL_q$  are the percentages of the variation of the active and reactive power losses, respectively.

**3.4.2. The index of the ‘voltage profile improvement’**

This index is defined as follows:

$$VPPII = \delta \cdot \left( \frac{VP_{With\ DG}}{VP_{Without\ DG}} - 1 \right) \times 100\% \tag{61}$$

where VPPII stands for the voltage profile improvement index. This illustrates the variation in the voltage profile after installing the DG resources. In addition, VP is the amount of voltage profile before and after installing these resources, and is calculated as follows:

$$VP = \sum_{i=1}^n V_i \tag{62}$$

where  $V_i$  is the value of each distribution bus in pu and  $\delta$  is defined as an index that identifies whether the voltage profile is in the allowable range or not.

$$\delta = \begin{cases} 1 & (0.95 < V_i < 1.05) \\ 0 & (V_i < 0.95 \text{ or } V_i > 1.05) \end{cases} \quad i = 1, 2, \dots, n \tag{63}$$

As shown in Eq. (63), if the voltage profile of even a single bus violates the allowable range, the value of  $\delta$ , and, consequently, the value of VPPII would be 0. The higher the value of VPPII, the more the voltage profile is improved.

**3.4.3. The index of the ‘short-circuit level’**

This index is calculated as follows:

$$ISC = \beta \cdot \left( \frac{I_{With\ DG}^{SC}}{I_{Without\ DG}^{SC}} - 1 \right) \times 100\% \tag{64}$$

where  $I^{SC}$  stands for the index of the short-circuit level and  $I_{Without\ DG}^{SC}$  and  $I_{With\ DG}^{SC}$  are the short-circuit levels of the network before and after installing the DG resources, respectively.

$$I^{SC} = \sum_{i=1}^n I_i^{SC} \tag{65}$$

Here,  $I_i^{SC}$  is the short-circuit current of each distribution bus before and after installing the DG resources. To identify whether the increase in the short-circuit level exceeds the tolerable amount of the CBs or not, the  $\beta$  coefficient is defined as follows:

$$\beta = \begin{cases} 1 & (I_i < I_{Switch,i}) \\ 0 & (I_i > I_{Switch,i}) \end{cases} \quad i = 1, 2, \dots, n \tag{66}$$

If the short-circuit level of all of the buses were in the tolerable range of the CBs,  $\beta$  would be equal to the value of 1. The value of the index of the increase in the short-circuit level can be calculated using Eq. (64) in this situation. If  $\beta$  is equal to 0, the short-circuit level of at least 1 bus has exceeded the tolerable range of the CBs. The index of the short circuit would be 0 in this situation.

4. Proposed method

4.1. Case study

To test the proposed method and formulation, the actual power network in Zanjan Province, in Iran, is selected for the simulation. Figure 4 depicts the single-line diagram of this network. In this network, the  $G_1$  bus is considered as the slack bus. The voltage value of this bus is 0.9625 pu.  $G_2$  is considered as the PV bus and the voltage value is 0.987 pu. The produced power by this bus is 110 MW. The active and reactive loads of this network are 288.55 MW and 93.53 MVar, respectively. Further information on this network is in [20].

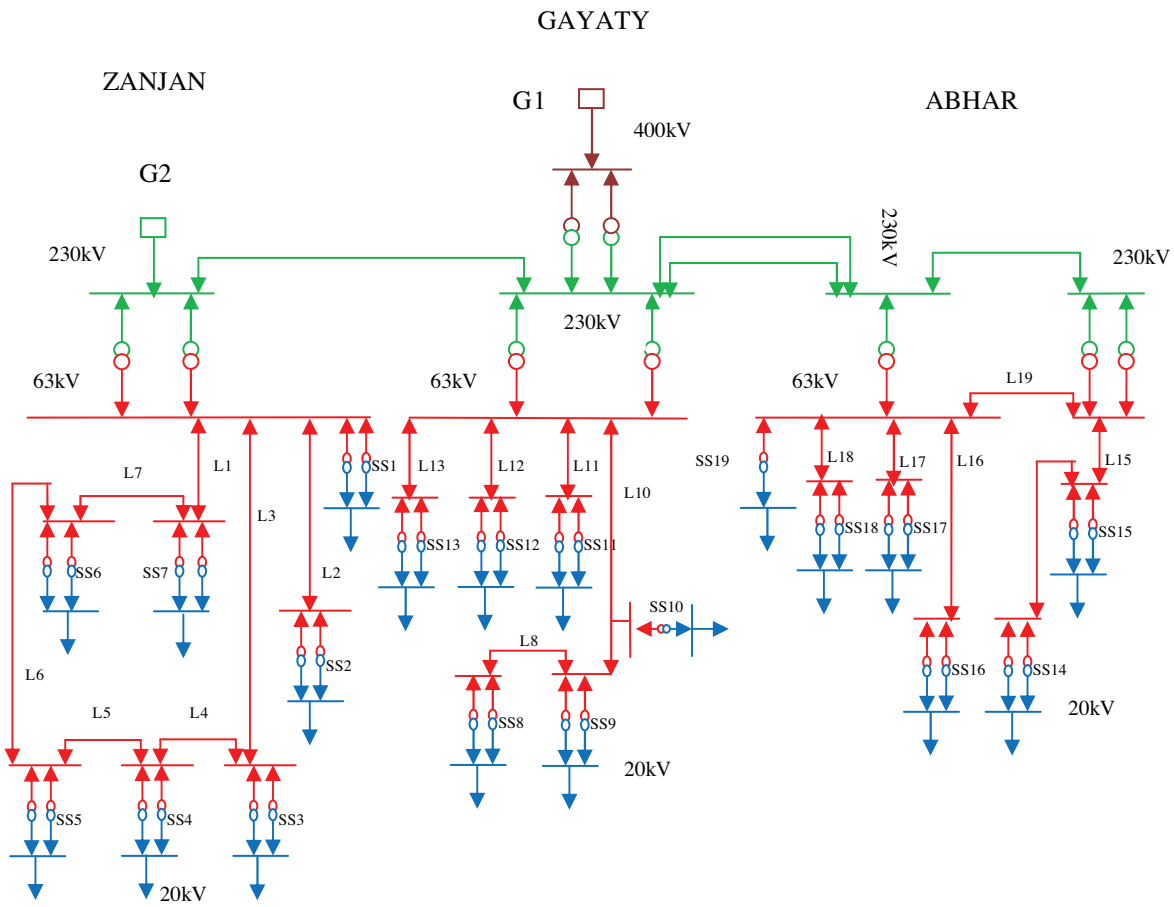


Figure 4. Single-line diagram of the Zanjan Province network.

4.2. Software

The proposed method and the sample network are simulated in the DIgSILENT Power Factory’s 14.0.523 program. The DIgSILENT program is an advanced software package for the simultaneous analyses of power network and control systems. This program is capable of calculating the load flow, short-circuit level, active and reactive losses of the network, and the parameters of the network. The main feature of this program is the ability of programming, which is called the DIgSILENT programming language.

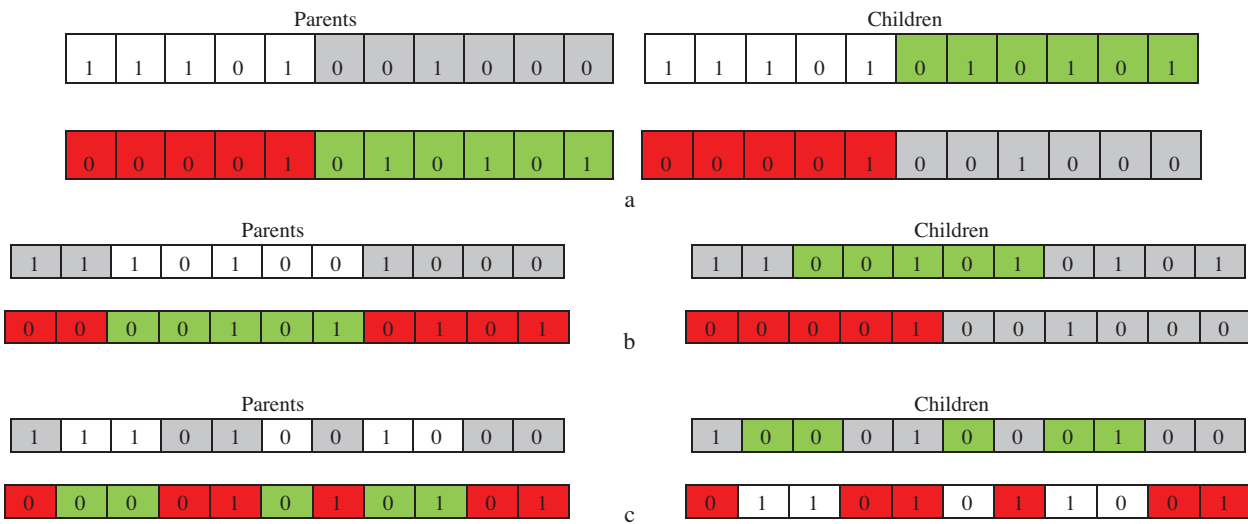
**4.3. Optimization technique**

After acquiring the OF formulation, it should be minimized. An optimization algorithm is required for this purpose.

In this paper, a genetic algorithm (GA) is used to optimize the objective function. To optimize a problem, using the GA, a population is required to be defined at the first step. This population is formed by the binary accidental quantization of chromosomes. In the next step, the produced population is applied to the objective function and the fitness of the chromosomes is obtained, using Eq. (67). Some of the best answers are chosen and a new generation is produced. This is done by genetic operators, which include crossover and mutation.

$$Fitness = \frac{1}{f} \tag{67}$$

In the crossover operator, 2 genes, that should be combined, are placed beside each other, and are divided from a specified point. Next, the sides that are placed in front of each other are combined together. The crossover genetic operator is implemented in 3 modes, which include the single point crossover, 2-point crossover, and uniform crossover. Each of these modes is depicted in Figure 5, where it can clearly be seen that the children strings carry all of their parent information integrated and without any changes.



**Figure 5.** Crossover operator: a) single point crossover, b) 2-point crossover, and c) uniform crossover.

Contrary to the crossover, each child is born from only one of the parents in the mutation genetic operator. The child is indeed created with the random changing of one of its parent chromosomes. Hence, a new generation is created with new chromosomes that did not exist in the parent. These new chromosomes are called mutation chromosomes. The genetic operator of the mutation is depicted in Figure 6.



**Figure 6.** Mutation operator.

To have both global and the fastest answers, both of these genetic operators are used in this paper. This part is implemented using the mfile programming section of the MATLAB software.

Since in this paper 3 capacities of 5, 10, and 15 MW are considered for DG resources and the number of 20 kV buses that DG can locate is 19, a string with 57 chromosomes ( $19 \times 3 = 57$ ) is required for each population. This string is depicted in Figure 7, where segment 1 is related to the buses that the 5 MW DGs are installed on. Segment 2 is related to the buses that the 10 MW DGs are installed on and segment 3 is related to the 15 MW DGs. In this string when a chromosome is 1, the proportional chromosomes in the other segments must be 0. The reason is that the aim is to install just 1 of the 5, 10, or 15 MW DGs on each bus. Therefore,

$$\text{Chromosomes which must be zero} = \text{Chromosomes which are one} + 19n \tag{68}$$

where  $n = \pm 1, \pm 2$ .

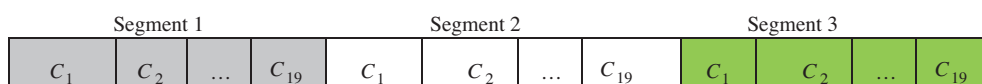


Figure 7. Coding for each population.

#### 4.4. The new algorithm and flowchart

To apply the proposed method, the following steps are considered.

1. The losses, short-circuit level, and voltage profile, in the absence of DGs, are calculated using the load flow program.
2. The proposed locations and capacities of the DGs, which are identified by the GA, are applied to the network.
3. If the constraints are satisfied the program will go to step 5, and, if not, it will go to step 4.
4. The answers are rejected and the program returns to step 2.
5. The initial weighting coefficients are calculated using Eqs. (50) to (58).
6. The best answers are identified, considering the OF and the indices.
7. The accuracy of the initial weighting coefficients is analyzed using Eqs. (69) to (73).
8. If the initial weighting coefficients are appropriate, go to the end, and, if not, go to step 7.

The flowchart of the proposed method is shown in Figure 8.

#### 5. Simulation results

Table 1 shows the capacities, locations, and number of DG resources obtained from applying the proposed algorithm and the selected weighting coefficients from the first stage.

As discussed in the formulation part, the proposed method of this paper is capable of deciding the optimal number, location, and capacity of the DG resources to obtain the most efficient installation. Accordingly, as illustrated in Table 1, to reduce the power loss, diminish the short circuit level, and improve the voltage profile simultaneously, 7 DG units with their identified locations and capacities are installed. The produced reactive power of these resources is assumed to be 0.8, which is the minimum value identified by electrical companies.



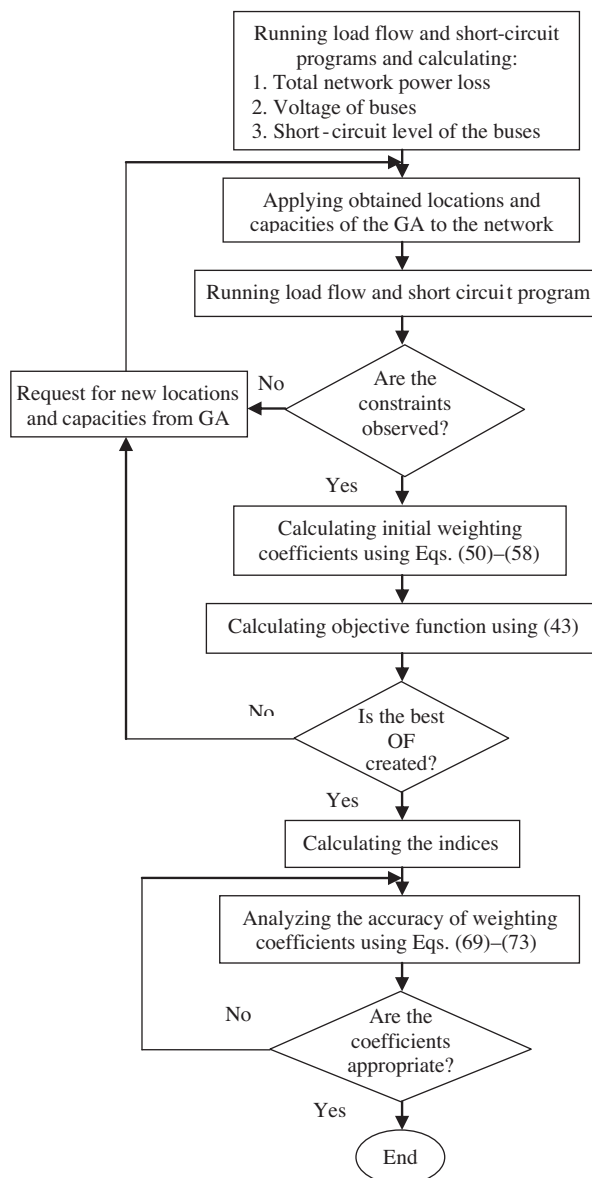
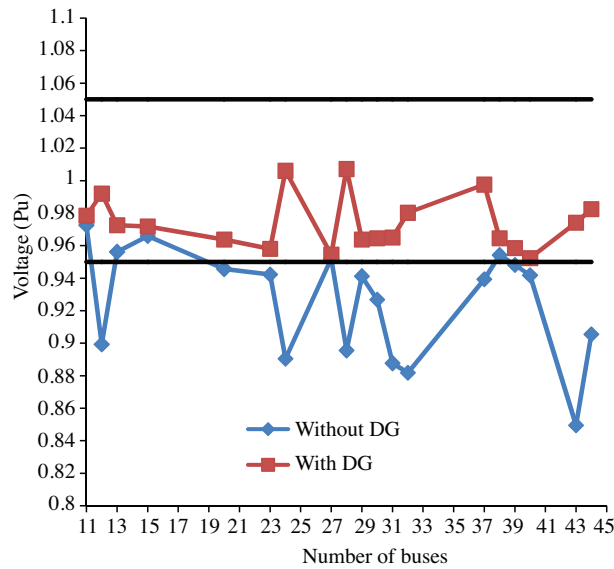


Figure 8. The algorithm flowchart.

Table 1. The obtained number, capacity, and location of the DG resources.

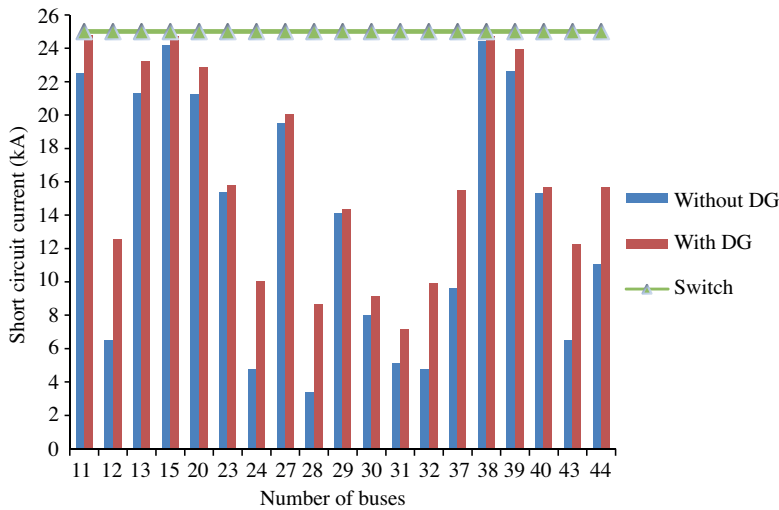
Location	Capacity (MW)	Capacity (MVar)	$CG_k$ (MVA)	Location	Capacity (MW)	Capacity (MVar)	$CG_k$ (MVA)
12	15	11.25	18.75	32	10	7.5	12.5
24	10	7.5	12.5	43	15	11.25	18.75
28	10	7.5	12.5	44	10	7.5	12.5
29	10	7.5	12.5				

As depicted in Figure 9, the voltage profile had an inappropriate condition before installing the DG resources and the voltage value of only 5 buses was in the allowable range. The voltage profile improved significantly after installing the DG resources and the voltage of all of the buses placed in the allowable range.



**Figure 9.** The voltage profile before and after installing the DG resources.

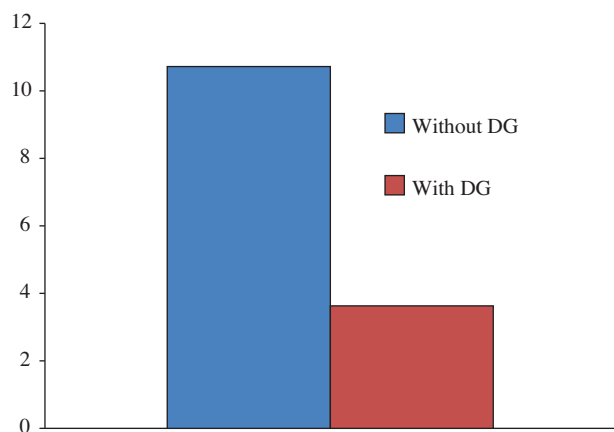
Figure 10 depicts the short-circuit levels of the buses before and after installing the DG resources. As is evident, the short-circuit levels of the buses increase after installing the DG units. The buses that the DG resources are installed on have the highest values of increase. Therefore, these buses should be considered more than the others. As illustrated, all of the increases have been in the tolerable range of the CBs, and, consequently, it is not essential to change them.



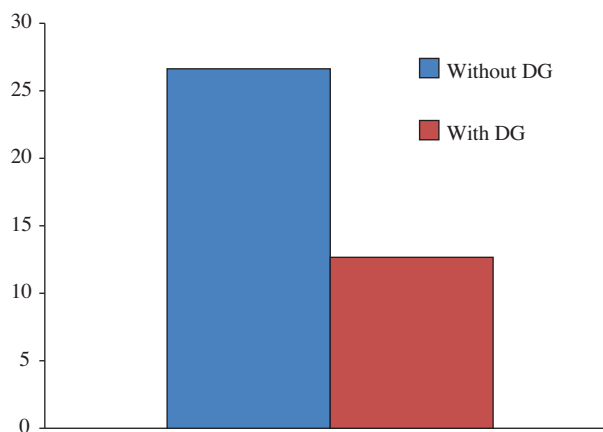
**Figure 10.** The short-circuit current of the distributed buses before and after installing the DG resources.

Figures 11 and 12 depict the active and reactive power losses before and after installing the DG resources.

As shown, both of the active and reactive losses have decreased after installing the DG resources. The active loss was 10.72 MW and it decreased to 3.63 MW after DG installation. This result also stands for the reactive loss. However, this factor was not inserted in the OF, but a reduction in the current of the lines, which was a result of installing the DG resources, has brought about the reduction of the reactive loss. The amount of reactive loss has decreased from 26.62 MVar to 12.67 MVar.



**Figure 11.** The total active loss of the network before and after installing the DG resources.



**Figure 12.** The total reactive loss of the network before and after installing the DG resources.

It is apparent from Table 2 that the values of the loading of the distribution substations have satisfied constraint 5. After installing the DG resources, all of the loadings of the distribution substations have decreased. This reduction is as a consequence of supplying a part of the demand load of the network, which is carried out by DGs. For instance, the loading of substation 12 was out of the permitted range in the absence of DGs and it was reduced by 89.01% after installing the DG resources. Having this amount of decrease, the loading of this substation became 34.05% and was placed in the permitted range. Therefore, the necessity of replacing the associated transformer was removed.

**Table 2.** Loading of the MV substations before and after installing the DG resources.

Number of bus	Loading before (%)	Loading after (%)	Number of bus	Loading before (%)	Loading after (%)
11	35.24	34.7	30	23.91	22.98
12	123.06	34.05	31	24.96	22.97
13	28.93	28.45	32	25.13	23.92
15	42.07	41.82	37	2.65	2.57
20	37.9	37.37	38	35.24	34.94
23	59.38	58.89	39	42.34	41.98
24	89.15	34.71	40	72.57	71.94
27	54.7	54.25	43	42.35	14.98
28	62.65	38.85	44	48.41	21.47
29	26.17	9.28			

The efficiency of the performed siting and sizing is evaluated, using the introduced indices. The values of these indices are presented in Table 3, where the voltage profile of the distribution network is seen to be improved by 4.88%. Having this improvement, all of the voltages of the distribution network buses were placed in the allowable range. In addition, the active and reactive losses of the network were reduced by 66.13% and 52.4%. Furthermore, the index of the short-circuit level increased by 18.8%.

**Table 3.** The indices of the sizing and siting.

$IL_P$	$IL_q$	$VP_{II}$	$ISC$
66.13	52.4	4.88	18.87

Since all of the buses of the distribution network have the capability of installing a DG unit, the different values of the indices are presented the situation where 1 to 19 DG units are installed. It is apparent from Figure 13 that by increasing the number of DG units, the index of the power loss is increased. However, in reality, it is not economical to increase the number of installed DG resources to more than an optimal value. On the other hand, this increasing results in the deterioration of the other parameters. Therefore, the other parameters should be considered while deciding the optimal number of DG resources.

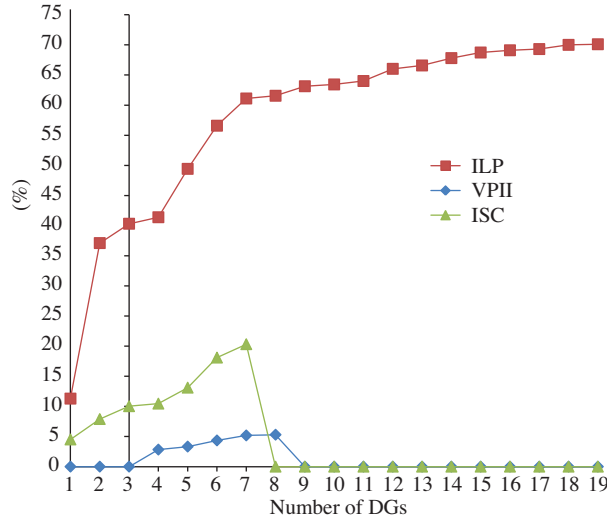


Figure 13. Different values of the indices for different numbers of DGs.

The value of VPII becomes 0 for less than 6 or more than 12 DG units. In these situations, the related constraint is violated. On the other hand, the value of  $I^{SC}$  becomes 0, when installing more than 8 DG units. This also shows that the associated constraint is violated. Consequently, for 6, 7, or 8 DG units, all of the constraints are satisfied and the optimal answer should be decided from these values. Considering the value of the OF, it is apparent that installing 7 DG units leads to the most appropriate value of the OF. These values are presented in Table 4.

Table 4. The values of the OF.

Number of DGs	OF value
6	0.053038
7	0.050109
8	0.0557997

### 6. Correcting the weighting coefficients

The considered weighting coefficients should be corrected according to the simulation results. For this purpose, Eq. (43) is ignored and Eq. (41) is used to calculate the OF. The weighting coefficients are therefore calculated as follows.

#### 6.1. Calculating the weighting coefficient of ‘a’

The weighting coefficient of  $\alpha$  is calculated according to the following equation:

$$a = \frac{1}{\frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}}} = \frac{P_{loss}^{withoutDG}}{P_{loss}^{withDG}} \tag{69}$$

Considering the simulation results, the weighting coefficient of  $\alpha$  equals the following value:

$$\begin{cases} P_{loss}^{withoutDG} = 10.72MW \\ P_{loss}^{withDG} = 3.63MW \end{cases} \rightarrow a = \frac{P_{loss}^{withoutDG}}{P_{loss}^{withDG}} = \frac{10.72}{3.63} = 2.95 \tag{70}$$

**6.2. Calculating the weighting coefficient of ‘ $b_k$ ’**

Since the relative importance of all of the buses is the same, and according to different sensitivities of the buses to voltage variations, the weighting coefficient of  $b_k$  is calculated as follows:

$$b_k = \frac{1}{(V_{bus,k}^{withDG} - 1)^2} \tag{71}$$

Table 5 is obtained using the simulation results and Eq. (71).

**Table 5.** The values of  $b_k$ .

The number of bus (k)	$V_{bus,k}^{withDG}$ (pu)	$(V_{bus,k}^{withDG} - 1)^2$	$b_k$
11	0.978408	$4.662145 \times 10^{-4}$	2144.94
12	0.991955	$0.64722 \times 10^{-4}$	15450.69
13	0.972559	$7.530085 \times 10^{-4}$	1328.01
15	0.971737	$7.987972 \times 10^{-4}$	1251.88
20	0.96	$16 \times 10^{-4}$	625
23	0.957975	$17.661006 \times 10^{-4}$	566.22
24	0.971818	$7.942251 \times 10^{-4}$	1259.09
27	0.959757	$16.19499 \times 10^{-4}$	617.47
28	1.009304	$0.865644 \times 10^{-4}$	11552.09
29	0.970098	$8.941296 \times 10^{-4}$	1118.41
30	0.964508	$12.596821 \times 10^{-4}$	793.85
31	0.964966	$12.273812 \times 10^{-4}$	814.74
32	0.980198	$3.921192 \times 10^{-4}$	2550.24
37	0.969412	$9.356257 \times 10^{-4}$	1068.8
38	0.964491	$12.608891 \times 10^{-4}$	793.09
39	0.958492	$17.229141 \times 10^{-4}$	580.41
40	0.952327	$22.727149 \times 10^{-4}$	440
43	0.974116	$6.699815 \times 10^{-4}$	1492.58
44	0.982476	$3.070906 \times 10^{-4}$	3256.37

**6.3. Calculating the weighting coefficient of ‘ $c_k$ ’**

$C_k$  is calculated as follows:

$$C_k = \frac{1}{\left(\frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}}\right)^2} = \left(\frac{i_{sc,k}^{withDG}}{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}\right)^2 \tag{72}$$

Table 6 is obtained using the simulation results and Eq. (72).

**Table 6.** The values of  $c_k$ .

The number of bus (k)	$i_{sc,k}^{withoutDG}$ (kA)	$i_{sc,k}^{withDG}$ (kA)	$c_k$	The number of bus (k)	$i_{sc,k}^{withoutDG}$ (kA)	$i_{sc,k}^{withDG}$ (kA)	$c_k$
11	22.5	24.8	116.26	30	8.01	8.991155	83.98
12	6.52	12.22194	4.59	31	5.11	7.007499	13.64
13	21.31	22.89313	209.11	32	4.76	9.714483	3.84
15	24.2	24.7	2440.36	37	9.62	10.38123	185.98
20	21.22	22.40446	357.79	38	24.43	24.7	8368.86
23	15.4	16.8	144	39	22.62	23.20336	1582.08
24	4.8	9.524158	4.06	40	15.34	15.44924	20000.9
27	19.49	20.01479	1454.56	43	6.54	11.58858	5.27
28	3.36	8.541929	2.72	44	11.09	17.67562	7.2
29	14.12	18.98747	15.22				

#### 6.4. Calculating the weighting coefficient of ‘ $d_k$ ’

$d_k$  is calculated as the following equation:

$$d_k = \frac{1}{\frac{CG_k}{S_{base}}} = \frac{S_{base}}{CG_k} \tag{73}$$

Table 7 is obtained using the simulation results and Eq. (73).

**Table 7.** The values of  $d_k$ .

The number of bus (k)	$CG_k$ (MVA)	$d_k$	The number of bus (k)	$CG_k$ (MVA)	$d_k$	The number of bus (k)	$CG_k$ (MVA)	$d_k$
11	0	-	27	0	-	37	0	-
12	18.75	5.33	28	12.5	8	38	0	-
13	0	-	29	12.5	8	39	0	-
15	0	-	30	0	-	40	0	-
20	0	-	31	0	-	43	18.75	5.33
23	0	-	32	12.5	8	44	12.5	8
24	12.5	8						

Since DG resources are not installed at all of the buses of the distribution networks,  $d_k$  can have any possible value in the locations where DG is not installed. It is worth mentioning that  $d_k$  should not be taken as a value of 0, for this may neglect the opportunity of placing DG at some locations. Since the maximum of  $d_k$  is 0.8, according to Table 7, this value is selected for the buses where no DG resource is installed. Thereby, Table 8 shows the weighting coefficients.

Using Eq. (56), these weighting coefficients can be normalized in Table 9.

The weighting coefficients, which are obtained from Table 9, are substituted in Eq. (41) and applied to the network to produce new locations and capacities for the DG resources. If the optimal answers of this stage conform to the previous stage, the selection of the weighting coefficients of the previous stage was appropriate. Otherwise, the selected weighting coefficients of the previous stage had been inappropriate and should be corrected.

**Table 8.** The values of the weighting coefficients.

The number of bus (k)	$a_k$	$b_k$	$c_k$	$d_k$	The number of bus (k)	$a_k$	$b_k$	$c_k$	$d_k$
11	2.95	2144.94	116.26	8	30	2.95	793.85	83.98	8
12	2.95	15450.69	4.59	5.33	31	2.95	814.74	13.64	8
13	2.95	1328.01	209.11	8	32	2.95	2550.24	3.84	8
15	2.95	1251.88	2440.36	8	37	2.95	1068.8	185.98	8
20	2.95	625	357.79	8	38	2.95	793.09	8368.86	8
23	2.95	566.22	144	8	39	2.95	580.41	1582.08	8
24	2.95	1259.09	4.06	8	40	2.95	440	20000.96	8
27	2.95	617.47	1454.5	8	43	2.95	1492.58	5.27	5.33
28	2.95	11552.09	2.72	8	44	2.95	3256.37	7.2	8
29	2.95	1118.41	15.22	8					

**Table 9.** The normalized weighting coefficients ( $\times 10^{-3}$ ).

The number of bus (k)	$a_k$	$b_k$	$c_k$	$d_k$	The number of bus (k)	$a_k$	$b_k$	$c_k$	$d_k$
11	1.3	944.01	51.17	3.52	30	3.32	893.19	94.49	9
12	0.19	999.17	0.3	0.34	31	3.51	970.71	16.25	9.53
13	1.91	857.84	135.08	5.17	32	1.15	994.23	1.5	3.12
15	0.8	338.05	658.99	2.16	37	2.33	844.42	146.93	6.32
20	2.97	628.94	360.04	8.05	38	0.32	86.46	912.35	0.87
23	4.09	785.14	199.68	11.09	39	1.36	267.05	727.91	3.68
24	2.32	988.21	3.19	6.28	40	0.14	21.51	977.96	0.39
27	1.42	296.43	698.31	3.84	43	1.96	991	3.5	3.54
28	0.26	998.81	0.24	0.69	44	0.9	994.46	2.2	2.44
29	2.58	977.13	13.3	6.99					

Tables 10 and 11 present the new values of the OF, number, capacities, and locations, which are obtained from applying the new weighting coefficients.

**Table 10.** The values of the OF with the new weighting coefficients.

Number of DGs	OF value
6	0.044576
7	0.044491
8	0.05173166

**Table 11.** Number, capacities, and locations of the DG resources using the new weighting coefficients.

Location	Capacity (MW)	Capacity (MVar)	$CG_k$ (MVA)	Location	Capacity (MW)	Capacity (MVar)	$CG_k$ (MVA)
12	15	11.25	18.75	32	10	7.5	12.5
24	10	7.5	12.5	43	15	11.25	18.75
28	10	7.5	12.5	44	10	7.5	12.5
29	10	7.5	12.5				

As presented in Table 10, and similar to the previous stage, all of the constraints are satisfied, placing 6, 7, or 8 DG resources. Consequently, the optimal answer should be calculated from one of these values. Apparently, the value of 0.044491 is assumed as the optimal OF, which is the result of placing 7 DG resources.

Comparing Tables 11 and 1, it is obvious that both of the situations have led to the same answer. Therefore, the selected weighting coefficients of the first stage have been decided appropriately, in order to obtain the optimal answer.

In Table 12, some random values are created by the DIGSILENT software and are designated to the weighting coefficients. These values are close to the weighting coefficients of the first stage. Clearly, all of the arbitrarily selected weighting coefficients bring about the answers that are identical to the answers of the first situation. Consequently, the most optimized answers for the OF are those that are created in the first situation.

**Table 12.** Locations, capacities, and number of DG units that were obtained by the random values.

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	Number of DGs	Location	Capacity (MW)	Capacity (MVar)
0.012839	0.954045	0.014681	0.018434	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.012470	0.96809	0.009827	0.009615	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.014517	0.947132	0.014311	0.024041	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.001983	0.963133	0.005875	0.029008	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.007093	0.965388	0.009872	0.017647	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.007485	0.961569	0.014456	0.016489	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.011685	0.968491	0.003766	0.015883	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.003856	0.938886	0.019544	0.037714	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.002403	0.978637	0.004096	0.014864	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.002243	0.987279	0.001065	0.009396	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.002792	0.987607	0.001595	0.008007	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.029376	0.896583	0.31810	0.042231	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5
0.028726	0.918211	0.022502	0.030562	7	12-24-28-29-32-43-44	15-10-10-10-10-15-10	11.25-7.5-7.5-7.5-7.5-11.25-7.5

## 7. Conclusion

In this paper, an innovative OF is proposed in order to identify the optimal location, capacity, and number of required DG resources. Furthermore, an algorithm is proposed to decide the best weighting coefficients according to the conditions of the network. The proposed method identifies different constraints of the network and minimizes the essential changes that should be performed after installing the DG units.

The proposed method is applied to the actual power network in Zanjan Province, in Iran, and the simulation results verify the efficiency of the method by obtaining the considered objectives, which included the reduction of the power losses, improvement of the voltage profile, and prevention of the excessive increase in the short-circuit level. In addition, some indices are presented and applied to evaluate the effectiveness of



the method. These indices are calculated and compared for different weighting coefficients and the comparisons carried out verify that the decided weighting coefficients are the best possible ones. The algorithm proposed in this paper is applicable to any siting and sizing procedure and in any network.

### References

- [1] M.M. Elnashar, R.E. Shatshat, M.M.A. Salama, "Optimum siting and sizing of a large distributed generator in a mesh connected system", *Electric Power Systems Research*, Vol. 80, pp. 690–697, 2010.
- [2] S. Ghosh, S.P. Ghoshal, S. Ghosh, "Optimal sizing and placement of distributed generation in a network system", *International Journal of Electrical Power & Energy Systems*, Vol. 32, pp. 849–856, 2010.
- [3] D.Q. Hung, N. Mithulananthan, R.C. Bansal, "Analytical expressions for DG allocation in primary distribution networks", *IEEE Transactions on Energy Conversion*, Vol. 25, pp. 814–820, 2010.
- [4] J.B.V. Subrahmanyam, C. Radhakrishna, "Distributed generator placement and sizing in unbalanced radial distribution system", *International Journal of Electrical and Electronics Engineering*, Vol. 3, pp. 746–753, 2009.
- [5] T. Gözel, M.H. Hocaoglu, "An analytical method for the sizing and siting of distributed generators in radial systems", *Electric Power Systems Research*, Vol. 79, pp. 912–918, 2009.
- [6] G.N. Koutroumpetis, A.S. Safigianni, "Optimum allocation of the maximum possible distributed generation penetration in a distribution network", *Electric Power Systems Research*, Vol. 80, pp. 1421–1427, 2010.
- [7] H.M. Khodr, M.R. Silva, Z. Vale, C. Ramos, "A probabilistic methodology for distributed generation location in isolated electrical service area", *Electric Power Systems Research*, Vol. 80, pp. 390–399, 2010.
- [8] N. Acharya, P. Mahat, N. Mithulananthan, "An analytical approach for DG allocation in primary distribution network", *International Journal of Electrical Power & Energy Systems*, Vol. 28, pp. 669–678, 2006.
- [9] V.H.M. Quezada, J.R. Abbad, T.G.S. Román, "Assessment of energy distribution losses for increasing penetration of distributed generation", *IEEE Transactions on Power Systems*, Vol. 21, pp. 533–540, 2006.
- [10] M. Mashhour, M.A. Golkar, S.M.M. Tafreshi, "Optimal sizing and siting of distributed generation in radial distribution network comparison: of unidirectional and bidirectional power flow scenario", *IEEE Bucharest Power Tech Conference*, pp. 1–8, 2009.
- [11] C. Wang, H.H. Nehrir, "Analytical approaches for optimal placement of distributed generation sources in power systems", *IEEE Transactions on Power Systems*, Vol. 19, p. 2068–2076, 2004.
- [12] S. Porkar, P. Poure, A. Abbaspour Tehrani-fard, S. Saadate, "Optimal allocation of distributed generation using a two-stage multi-objective mixed-integer-nonlinear programming", *European Transactions on Electrical Power*, Vol. 21, pp. 1072–1087, 2010.
- [13] M. Ahmadigorji, A. Abbaspour, A.R. Ghahnavieh, M. Fotuhi-Firuzabad, "Optimal DG placement in distribution systems using cost/worth analysis", *World Academy of Science, Engineering and Technology*, Vol. 49, pp. 746–753, 2009.
- [14] R.K. Singh, S.K. Goswami, "Optimum allocation of distributed generations based on nodal pricing for profit, loss reduction, and voltage improvement including voltage rise issue", *International Journal of Electrical Power & Energy Systems*, Vol. 32, pp. 637–644, 2010.
- [15] J.H. Teng, Y.H. Liu, C.Y. Chen, C.F. Chen, "Value-based distributed generator placements for service quality improvements", *International Journal of Electrical Power & Energy Systems*, Vol. 29, pp. 268–274, 2007.
- [16] G. Celli, E. Ghiani, S. Mocci, F. Pilo, "A multi-objective evolutionary algorithm for the sizing and siting of distributed generation", *IEEE Transactions on Power Systems*, Vol. 20, pp. 750–757, 2005.
- [17] W. El-Khattam, K. Bhattacharya, Y. Hegazy, M.M.A. Salama, "Optimal investment planning for distributed generation in a competitive electricity market", *IEEE Transactions on Power Systems*, Vol. 19, pp. 1674–1684, 2004.

- [18] P. Chiradeja, R.G. Ramakumar, "An approach to quantify the technical benefits of distributed generation", *IEEE Transactions on Energy Conversion*, Vol. 19, pp. 764–773, 2004.
- [19] C.L.T. Borges, D.M. Falcao, "Optimal distributed generation allocation for reliability, losses, and voltage improvement", *International Journal of Electrical Power & Energy Systems*, Vol. 28, pp. 413–420, 2006.
- [20] F. Razavi, S.A. Hosseini, M. Karami, A.A. Ghadimi, S.S. Karimi Madahi, "Determining the optimal capacity and place of DGs using GA algorithm: voltage profile improvement and loss reduction", *The 8th Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology Association, Thailand - Conference*, pp. 848–851, 2011.
- [21] L.F. Ochoa, A. Padilha-Feltrin, G.P. Harrison, "Evaluating distributed generation impacts with a multi objective index", *IEEE Transactions on Power Delivery*, Vol. 21, pp. 1452–1458, 2006.