

## Optimal Capacity, Location and Number of Distributed Generation at 20 kV Substations

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**Abstract:** Restructuring of the electrical industry and the impetus for using new energies has brought about the increasing applications of Distributed Generation (DG). Hence, these resources are expected to play a crucial role in near future of this industry. Installing DG units in distribution network may result in positive impacts such as, voltage profile improvement and loss reduction and negative impacts such as, the increase in the short-circuit level. These impacts depend on the exploitation methods, installation place and the capacity of these resources. Therefore, finding the optimal place and capacity of DG resources are of the crucial importance. Accordingly, this paper is aimed to find the optimal place and capacity of DG resources, in order to improve the technical parameters of network, including power losses, voltage profile and short-circuit level. Identifying the optimal number of the DG resources one of the advantages of the proposed method is which provides a balance between the number of installed DGs and the maximum technical acquirable advantages of them. Another advantage of proposed method is capability of simultaneous deciding of optimal capacity and location of a number of DGs. Genetic Algorithm is used to minimize the Objective Function and to find the best answers during the investigation. Finally, the proposed algorithm is tested on distribution network of Zanjan Province in Iran and the simulation results are presented and discussed.

**Key words:** DG Optimal Location, DG Optimal Size, Distributed Generation, Objective Function, Genetic Algorithm.

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### INTRODUCTION

DG is defined as an electrical power resource which directly connected to the network (Mohab M. Elnashar, 2009). Application of DG is increasing rapidly due to the limitations of fuel cells and the new environmental view points. In addition, the governments of the developing and under-development countries are supporting the DG for they can supply the required electrical energy of their increasing customers. Installing DGs at the network buses have a direct impact on the flowing power and the voltage of the network. This impact depends on many different factors and may be positive or negative (Koutroumpetzis, 2010). The positive impacts of installing DG resources include increasing the power quality, improving the voltage profile, reducing the power loss, decreasing the requirements of installing new transmission lines and deferring the necessity of improving the capacity of substations (Gozel, 2009; Subrahmanyam, 2009). On the other hand, the main adverse impact of installing DG is the increase in short circuit level of the network (Koutroumpetzis, 2010).

The studies show that if the capacity and location of DGs are not identified appropriately, not only the network parameters are not improved, but also they are deteriorated (Mendez Quezada, 2006; Acharya, 2006). Thereby, two of the most important factors of DG plans are identifying the capacity and location of these resources (Sudipta Ghosh, 2010). The place and capacity of DGs can be decided according to the improvement of one or more parameter, in order to increase the efficiency and decrease the adverse effects of installing them. However, siting and sizing DGs, with the aim of improving a single parameter, enhances the considered parameter significantly, but may have negative impact on other parameters of the network. On the other hand, siting and sizing DG with the purpose of enhancing some of the parameters of the network will result in improvement of the considered parameters. Considering the impacts of different parameters is an important issue, while having a multi- objective siting and sizing. On the other hand, increasing the number of installed DGs at the network will causes some extra costs due to the DGs installing and their service costs. Hence, there should be a balance between the imposed costs and the improvement of the considered parameters of the network. This balance can be created by deciding the optimal number of DGs. Therefore, the technical parameters of the network can be improved significantly, while the imposed costs are not illogically.

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Over the past few decades, several studies have been done for the purpose of siting and sizing DG resources. Ref., (Mohab M. Elnashar, 2009) decides the optimal place and capacity of DG, in order to improve voltage profile, reduce power loss and diminish the short-circuit level of the buses. The proposed formulation of this paper is based on a heuristic method. The objective of siting and sizing of (Koutroumpezis, 2010) is satisfying different constraints and reaching to the maximum penetration level of DG. Considered constraints of this paper include the flowing power of the terminals, the capacity of transformers, voltage profile and short-circuit level.

In (Gozel, 2009) the place and capacity are found, according to the reduction of the power loss. An analytical method is used in this paper which does not require calculating the admittance and impedance matrices. Voltage profile is the only considered constraint of this paper. Ref. (Acharya, 2006) has decided the optimal capacity and location of DG based on an exact formulation and using consecutive load-flows and the power loss sensitivity. In this method,  $Z_{bus}$  and  $Y_{bus}$  matrices are not required to be calculated. But, this method is not applicable in real network due to the size, complexity and special conditions of the real distribution networks. Siting and sizing DG resources was done in (Hung, 2010) analytically so as to reduce energy losses and to determine a suitable power factor. As a result of not using any optimal method, this model increased the speed convergence to find a suitable answer. The work of (Atwa, 2010) has found the optimal place and capacity of DG units with renewable energies such as wind, solar and biomes energy. The aim of this work was reducing the power losses. Modeling the load and using a probability function, this paper has covered all the situations where the DG may encounter in the network. Authors in (Soo-Hyoung Lee, 2009) have decided the optimal location and capacity of DG units, in order to diminish the power losses. For this aim the Kalman Filter Algorithm is used. The work of (R.A. Jabr, 2009) proposed a trade-off between the maximum capacity of DG and the minimum system losses. In addition, an Ordinal optimization technique which is based on some probability lows is used. Authors in (Mashhour, 2009) proposed a formulation, in order to decide the optimal place and capacity of DG to reduce the power loss in two situations. In the first situation, DG units supply the required power of the loads (unidirectional) and in the second situation, not only the DG units supply the required power of the loads but also they inject some power to the upper network (bidirectional). Injecting power to upper network is usually prevented by the electrical companies due to the protective problems that the cause. In (Wang, 2004) an analytical method is proposed, to decide the optimal place and capacity of DG units in radial networks. This paper is aimed to minimize the power loss. The proposed method of this work is based on using  $Z_{bus}$  and  $Y_{bus}$  matrices. (Singh, 2009) presented a genetic algorithm-based method to determine optimal location and size of the DG to be placed in radial, as well as networked, systems with an objective to minimize the power losses. (Ochoa, 2006) and (Pathomthat Chiradeja, 2006) presented some indices such as, power loss index, voltage profile index and short-circuit index, in order to evaluate the efficiency of installed DG units. Violating the constraints could not be identified using the proposed indices of these works.

This paper proposes a method to decide the optimal location and capacity of DG units in MV substations. The proposed method conforms to the purposes of the most of the electrical companies of Iran Country. These purposes includes, the reduction of power losses, improvement of voltage profile and reduction of the short-circuit level. In addition, since one of the major problems of installing these resources is associated with the optimal number of required DG units, this parameter is identified in this paper and for the test network. To acquire the considered purposes, the DG units are installed simultaneously.

Due to the changes in the network parameters, generally, some network components must be replaced after installing the DG units. To minimize these changes, some constraints are presented to identify some limitations for the changes of network parameters. Finally, the proposed method is tested on actual power network of Zanjan Province in Iran and simulation results are presented and discussed.

#### ***Problem Statement:***

The most important defects of the previous works are presented in this section. This paper is aimed to enhance these defects. They include, sizing and siting DG having a single objective and ignoring the selection of the optimal number of DG units.

#### ***Sizing and Siting DG Having A Single Objective:***

One of the major defects of previous works of siting and sizing is considering only one objective for the problem. In these studies the impact of installing DG resources on the other parameters of the network is not considered. To clarify the problem, two radial networks, with and without DG are considered as illustrated in the Figure 1 and Figure 2. The considered load of the network is a balanced 3-phases load with the constant power of  $S_L$  which is located at the end of the feeder.

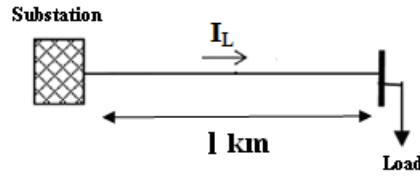


Fig. 1: The considered network without DG.

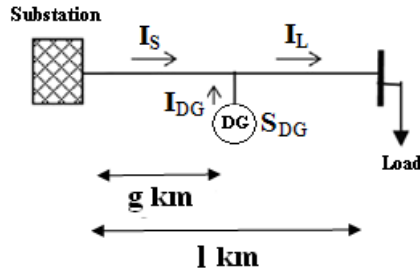


Fig. 2: The considered network with DG.

**The Impact Of DG On Power Loss:**

Installing DG units at the network buses may bring about the reduction or intensification of power losses (Mithulananthan, 2004; Carmen L.T., 2006).

The amount of power loss in the absence of DG can be calculated by the following equation.

$$P_{loss}^{Without DG} = 3 \cdot r \cdot l \cdot |I_L|^2 \tag{1}$$

where  $P_{Loss}^{Without DG}$  is the power loss in the absence of DG,  $r$  is the resistance of the line per length, and  $V_L$  is the voltage of customer terminal.

The amount of power loss in the presence of DG can be calculated by the following equation.

$$P_{loss}^{With DG} = P_{loss1} + P_{loss2} \tag{2}$$

where  $P_{Loss}^{With DG}$  is the power loss in the presence of DG,  $P_{Loss1}$  is the power loss from the substation to DG location, and  $P_{Loss2}$  is the power loss from DG location to the load location.

$$P_{loss}^{With DG} = 3rg|I_S|^2 + 3(l-g)r|I_L|^2 \tag{3}$$

Therefore, the variation of power loss after installing DG is obtained using (4).

$$\Delta P_{loss} = \frac{rg \cdot (|S_L^* \cdot V_{DG}^* - S_{DG}^* \cdot V_L^*|^2 - |S_L^* \cdot V_{DG}^*|^2)}{3(|V_L^*| |V_{DG}^*|)^2} \tag{4}$$

It can be concluded from (4) that identifying an appropriate capacity for DG unit plays a pivotal role in reducing the power loss. On the other hand, if the considered capacity was inappropriate, it may increase the power loss.

**The Impact of DG on Voltage Profile:**

Installing DG on distribution feeders may result in an improvement or deterioration of voltage profile of the customer terminal. The voltage profile of the customer in the absence of DG is calculated using (5).

$$V_L^{Without DG} = V_S - (r + jx)l \cdot I_L \tag{5}$$

where  $V_S$  is the voltage of the substation and is assumed to be 1 pu. The voltage of the customer in the existence of DG is calculated, using (6).

$$V_L^{With DG} = V_S - (r + jx) \cdot g \cdot I_S - (r + jx) \cdot (l - g) \cdot I_L \quad (6)$$

Comparing (5) and (6) the variation of the voltage of the customer, after installing DG is calculated.

$$\Delta V_L = (r + jx) \cdot g \cdot (I_L - I_S) = (r + jx) \cdot g \cdot I_{DG} \quad (7)$$

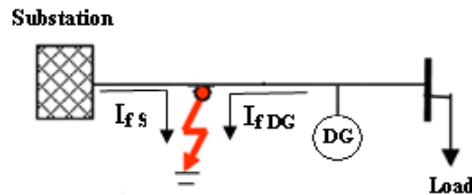
The amount of increase in voltage of customer, after installing DG is obtained.

$$|\Delta V_L| = \frac{\sqrt{r^2 + x^2} \cdot g \cdot |S_{DG}|}{3|V_{DG}|} \quad (8)$$

It is apparent from (8) that ignoring the amount of produced power of DG may result in too much increase in voltage profile and thereby, it may bring about exceeding the allowable limit.

**The Impact Of DG In Short-Circuit Level:**

DG unit is almost always placed in parallel with the network. Hence, the calculated impedance from fault point of view diminishes and the fault current level increases. Increasing the DG unit capacity, the equivalent impedance of DG unit diminishes and the short-circuit level of the network changes (Nguyen Hoang Viet, 2007).



**Fig. 3:** Increase in short-circuit level in the presence of DG.

The short-circuit level in the absence of DG equals to:

$$|I_{Fault}| = |I_{fs}| \quad (9)$$

Therefore, the fault current equals to the flowing current of the substation, before installing DG and its value changes into (10) after installing DG.

$$|I_{Fault}| = |I_{fs} + I_{fDG}| \quad (10)$$

If after installing DG resources, the increase in the short-circuit level exceeds the tolerable range of the circuit-breakers (CBs), they must be substituted by suitable CBs. This imposes an excessive cost on electrical companies. Therefore, the short-circuit level of the buses should be considered while siting and sizing DG resources.

According to the preceding discussions, the place and capacity of DG units is effective on the technical parameters of the network. Therefore, the problems which are caused by installing the DG units can be avoided and the maximum efficiency of these resources is acquirable, considering the technical parameter of sizing and siting and having the optimal sizing and siting.

**Installing an Inappropriate Number of DG:**

Installing inappropriate number of DG brings about many different problems. It may increase the short-circuit level too much or deteriorate the voltage profile. Hence identifying the optimal number of DG units is an issue of a great importance which may have a crucial importance on the technical parameters of the network such as voltage profile, short-circuit level and etc.

**New Method:**

This paper proposes a siting and siting method for DG units in MV substations according to the requirements of the electrical companies of Iran Country. Therefore the OF is composed of the most important parameters of these companies. These parameters includes, power losses of the network, voltage profile of the distribution buses, the short-circuit level of the distribution buses, appropriate number and optimal location and capacity of DG units. In this section the OF is presented. Furthermore, the considered constraints of sizing and siting and the considered indices are presented.

**The Objective Function:**

To minimize a function which is consisted from some parameters, the general function can be written as a summation of those parameters.

$$f = f_1 + f_2 + \dots + f_N = \sum_{i=1}^N f_i \tag{1}$$

where N is the number of factors which affect the OF. It is assumed to be 5 in this paper. Each of these factors is presented in the following.

**Parameter Of ‘Total Power Loss Of The Network’:**

Power loss of the network is calculated in (12).

$$f_1 = f(P_{loss}) = P_{loss} \tag{2}$$

where  $P_{loss}$  is the total power loss of the network. Normalizing the  $P_{loss}$  and considering  $\alpha$  for weighting coefficient, the final function of  $f_1$  is acquired from (13).

$$f_1 = \alpha \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} \tag{3}$$

**Parameter of ‘Voltage Profile’:**

The voltage profile parameter depends on the voltage of the buses. This parameter is defined as a variation of voltage from the ideal value of 1 pu.

$$v = V_{bus}^{withDG} - 1(Pu) \tag{4}$$

Since the function of v is a pu value, it is not required to normalize it. To identify the voltage variation of all buses from 1 pu, the V vector is defined as (15).

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \tag{5}$$

where n is the number of the distribution network buses.

$$v_k = V_{bus,k}^{withDG} - 1, \quad k = 1, 2, \dots, n \tag{6}$$

where  $v_k$  is the voltage variation of kth bus from 1 pu. The function of  $f_2$  is acquired, calculating the voltage variation of each bus from 1 pu.

$$f_2(V_{bus}) = V^T B V \tag{7}$$

where B is the matrix of weighting coefficients and is defined as (18).

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix}_{n \times n} \quad (8)$$

Therefore,  $f_2$  equals:

$$f_2(V_{bus}) = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}^T \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \quad (9)$$

Simplifying (19), (20) is obtained as follows.

$$f_2(V_{bus}) = \sum_{i=1}^n \sum_{j=1}^n b_{ij} v_i v_j \quad (10)$$

Replacing (16) in (20) results in (21).

$$f_2(V_{bus}) = \sum_{k=1}^n b_{kk} (V_{bus,k}^{withDG} - 1)^2 + \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n b_{ij} (V_{bus,i}^{withDG} - 1)(V_{bus,j}^{withDG} - 1) \quad (11)$$

The voltage of two different buses has no impact on each other, thus the value of  $b_{ij}$  for  $i \neq j$  is assumed to be zero. The voltage profile, thereby, equals the following equation.

$$f_2(V_{bus}) = \sum_{k=1}^n b_k (V_{bus,k}^{withDG} - 1)^2 \quad (12)$$

**The Parameter Of ‘Short-Circuit Level’:**

The function of  $w$  is defined as (23). This function shows the difference of the short-circuit current in the presence and absence of DG.

$$w = i_{sc}^{withDG} - i_{sc}^{withoutDG} \quad (13)$$

Normalizing  $w$  results in (24).

$$w = \frac{i_{sc}^{withDG} - i_{sc}^{withoutDG}}{i_{sc}^{withDG}} \quad (14)$$

$W$  vector is defined as (25) to show the variation of short-circuit level of all the buses in the absence and existence of DG.

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad (15)$$

Each element of  $W$  is calculated by (26).

$$w_k = \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}}, \quad k = 1, 2, \dots, n \quad (16)$$

According to the foregoing equations, the function of short-circuit level is calculated as follow.

$$f_3(i_{sc}) = W^T C W \quad (17)$$

where C is the matrix of weighting coefficients and is defined as follow.

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}_{n \times n} \quad (18)$$

$f_3$  is calculated in (29) with respect to the (28).

$$f_3(i_{sc}) = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}^T \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \quad (19)$$

Simplifying the foregoing equation, the parameter of short-circuit level is obtained in (30).

$$f_3(i_{sc}) = \sum_{i=1}^n \sum_{j=1}^n c_{ij} w_i w_j \quad (20)$$

Substituting (26) in (30), results in the following equation.

$$f_3(i_{sc}) = \sum_{k=1}^n c_{kk} \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + \sum_{i=1}^n \sum_{\substack{j=1 \\ j \neq i}}^n c_{ij} \left( \frac{i_{sc,i}^{withDG} - i_{sc,i}^{withoutDG}}{i_{sc,i}^{withDG}} \right) \left( \frac{i_{sc,j}^{withDG} - i_{sc,j}^{withoutDG}}{i_{sc,j}^{withDG}} \right) \quad (21)$$

The difference of short-circuit level of two different buses has no impact on each other, thus the value of  $c_{ij}$  for  $i \neq j$  is assumed to be zero. The parameter of the short-circuit level, therefore, equals the following equation.

$$f_3(i_{sc}) = \sum_{k=1}^n c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 \quad (22)$$

**The Parameter of ‘Capacity of DG Units’:**

Installing and exploiting some DG resources with a small capacity is proven to be more effective than a single DG with a big capacity (Mashhour, 2009; Singh, 2009). Therefore, one of the objectives of this paper is sizing and siting a number of DG units with small capacity. To reach this purpose,  $f_4$  is defined as follow.

$$f_4(CG) = \sum_{k=1}^n d_k CG_k \quad (23)$$

where  $CG_k$  is the capacity of installed DG on kth bus and in MVA and  $d_k$  is the related weighting coefficient. Dividing (33) into  $S_{base}$  normalizes the  $f_4$ .  $S_{base}$  is the base value of actual power of the network.

$$f_4(CG) = \sum_{k=1}^n d_k \frac{CG_k}{S_{base}} \tag{24}$$

Using this equation, the capacity of DG units is optimized.

**The Parameter of ‘Number of DG Units’:**

This parameter is defined as the following equation

$$f_5(n_{DG}) = n_{max} - n_{DG} \tag{25}$$

where  $n_{max}$  is the maximum number of DG units which can be installed in the network. Since the relative importance of all the buses is equal during siting and sizing, and DG can be placed at all the buses,  $n_{max}$  is equal to the number of 20 KV buses.  $n_{DG}$  is the number of installed DG units. Using this equation, a balance can be created between the number of DG resources and improvement of the technical parameters.

**Summarize:**

Substituting  $f_1, f_2, f_3, f_4$  and  $f_5$  by their obtained value will results in the following equation.

$$f = a \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + \sum_{k=1}^n b_k (V_{bus,k}^{withDG} - 1)^2 + \sum_{k=1}^n c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + \sum_{k=1}^n d_k \frac{CG_k}{S_{base}} + e(n_{max} - n_{DG}) \tag{26}$$

Simplifying (36), (37) is obtained as follow.

$$f = \sum_{k=1}^n \left( \frac{a}{n} \right) \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + \sum_{k=1}^n b_k (V_{bus,k}^{withDG} - 1)^2 + \sum_{k=1}^n c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + \sum_{k=1}^n d_k \frac{CG_k}{S_{base}} + \sum_{k=1}^n \left( \frac{e}{n} \right) (n_{max} - n_{DG}) \tag{27}$$

$$f = \sum_{k=1}^n \left( \left( \frac{a}{n} \right) \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + b_k (V_{bus,k}^{withDG} - 1)^2 + c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + d_k \frac{CG_k}{S_{base}} + \left( \frac{e}{n} \right) (n_{max} - n_{DG}) \right) \tag{28}$$

This equation can be rewritten as follows.

$$f = \sum_{k=1}^n \left( a_k \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + b_k (V_{bus,k}^{withDG} - 1)^2 + c_k \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + d_k \frac{CG_k}{S_{base}} + e_k (n_{max} - n_{DG}) \right) \tag{29}$$

Since all the buses are as important as each other, the weighting coefficients of all the buses are assumed to be equal. Therefore,

$$\begin{aligned} a_1 &= a_2 = \dots = a_n = a \\ b_1 &= b_2 = \dots = b_n = b \\ c_1 &= c_2 = \dots = c_n = c \\ d_1 &= d_2 = \dots = d_n = d \\ e_1 &= e_2 = \dots = e_n = e \end{aligned} \tag{30}$$

Consequently, the considered OF is as follow.

$$f = \sum_{k=1}^n \left( a \frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} + b (V_{bus,k}^{withDG} - 1)^2 + c \left( \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} \right)^2 + d \frac{CG_k}{S_{base}} + e (n_{max} - n_{DG}) \right) \tag{31}$$



**Constrains:**

Constraints are an issue of a great importance in optimization procedures. Indeed, an optimal answer is an answer which satisfies all the constraints of optimization problem. In this paper, the following technical constraints are considered while siting and sizing.

**Constraint of ‘Voltage of The Buses’:**

The variation range of all the distribution buses should be within a specified limit (Mohab M. Elnashar, 2009; Razavi, 2011).

$$V_{\min} < V_{bus,k}^{WithDG} < V_{\max} \tag{32}$$

where  $V_{\min}(0.95(\text{pu}))$  and  $V_{\max}(1.05(\text{pu}))$  are the limits of allowable voltages. Since the values of the voltages were inserted in OF as square of their variations from 1 pu the following equation stands.

$$0.95 < V_{bus,k}^{withDG} < 1.05 \rightarrow |V_{bus,k}^{withDG} - 1| < 0.05 \rightarrow (V_{bus,k}^{withDG} - 1)^2 < 0.0025 \tag{33}$$

**Constraint of ‘Short-Circuit Current in The Existence of DG Units’:**

The increase in the short-circuit level, in the presence of DG, must be within the tolerable range of the CBs. Thus, (44) should stand (Mohab M. Elnashar, 2009).

$$I_{sc,k}^{withDG} < \text{Short circuit level of the currently installed protective devices} \tag{34}$$

**The Summation of ‘DG Capacities’:**

The maximum of summation of produced active power of DG units must not exceed the load demand of the network (Hung, 2010).

$$\sum_{k=1}^{NDG} CG_k \leq P_{load} \tag{35}$$

where  $N_{DG}$  is the number of DG units. Observing this constraint prevents bidirectional power flow.

**The Constraint of ‘The Minimum Power Factor of The DG Units’:**

Synchronous generators are capable of producing active and reactive power, simultaneously (Hung, 2010). Since electrical companies are more interested in operating in upper power factors, this constraint should be considered while sizing and siting. The mentioned constraint is defined as follow.

$$0.8 \leq PF_{DG,k} \leq 1, \quad k = 1, 2, \dots, n_{DG} \tag{36}$$

where  $PF_{DG,k}$  is the power factor of each DG unit. In this paper the power factors are assumed to be 0.8.

**Constraint of Loading of ‘Distribution Substations’:**

Installing DG units should not increase the loading of transformers more than their allowable range (Koutroumpetzis, 2010; Mashhour, 2009). Otherwise, the electrical companies have to replace the overloaded transformers.

$$Trans_{Loading} \leq 90\% \tag{37}$$

Constraints can be divided into soft and hard constraints. Hard constraints are those which violating them is not permitted and soft constraints are those which can be violated to some extend (Mashhour, 2009). To prevent any changes in distribution network as a result of installing DG, all the considered constraints of this paper are assumed to be hard.

**Identifying The Weighting Coefficients:**

Weighting coefficients are generally selected by the designer to identify the relative importance of each parameter. The designer decides them according to the requirements of each network and the objectives of siting and sizing (Mohab M. Elnashar, 2009). In this paper the following method is used to identify the weighting coefficients.

**The Weighting Coefficient of ‘Power Loss’:**

The power losses can be reduced with the appropriate siting and sizing of DG resources. This value can be increased down to zero, but it is not economical due to the cost of DG units. It is proven that assuming  $\frac{P_{loss}^{withDG}}{P_{loss}^{withoutDG}} = 0.5$ , the weighting coefficient of power loss can have the most economical value. The weighting coefficient of  $a$  is therefore calculated as (48).

$$a = \frac{1}{0.5} = 2 \tag{38}$$

**The Weighting Coefficient of ‘Voltage Profile’:**

$b$  stands for the weighting coefficient of voltage and with respect to the (43) it can be obtained from the following equation.

$$b = \frac{1}{0.0025} = 400 \tag{39}$$

**The Weighting Coefficient of ‘Short-Circuit Level’:**

According to different analyses in the actual power networks, the short-circuit level can be increased up to 1.5 times, after installing DG resources. Therefore:

$$i_{sc,k}^{withDG} = 2.5i_{sc,k}^{withoutDG} \rightarrow \frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}} = 0.6 \rightarrow \left(\frac{i_{sc,k}^{withDG} - i_{sc,k}^{withoutDG}}{i_{sc,k}^{withDG}}\right)^2 = 0.36 \tag{40}$$

As a consequence, the weighting coefficient of  $c$  equals to:

$$c = \frac{1}{0.36} = 2.78 \tag{41}$$

**The Weighting Coefficient of ‘Capacity of DG Units’:**

The assumed capacity of DG units are 5, 10 and 15 MW, the power factor is assumed to be 0.8 lag and  $S_{base}$  is assumed to be 100 MVA in this paper. Therefore, the following equation stands.

$$CG_k = \frac{P_{DG,k}}{\cos\phi} = \frac{10}{0.8} = 12.5MVA \rightarrow \frac{CG_k}{S_{base}} = \frac{12.5}{100} = 0.125 \tag{42}$$

Therefore the weighting coefficient of  $d$  equals to:

$$d = \frac{1}{0.125} = 8 \tag{43}$$

**The Weighting Coefficient of ‘Number of DG Units’:**

According to the simulation results on the test network, in the absence of  $f_5$  function, DG units are installed at 7 buses which includes buses 12, 24, 28, 32, 29, 43 and 44.

Therefore, the weighting coefficient of  $e$  is defined as follows.

$$e = \frac{1}{n_{max} - n_{DG}} = \frac{1}{19 - 7} = 0.0833 \tag{44}$$

The obtained weighting factors should be normalized, to have the best answers.

$$\begin{cases} a = \frac{a}{a+b+c+d+e} \\ b = \frac{b}{a+b+c+d+e} \\ c = \frac{c}{a+b+c+d+e} \\ d = \frac{d}{a+b+c+d+e} \\ e = \frac{e}{a+b+c+d+e} \end{cases} \quad (45)$$

Normalizing the weighting coefficients, the summation of them turns out to be 1 (Mohab M. Elnashar, 2009).

$$a+b+c+d+e=1 \quad (46)$$

As a result, the initial values of the weighting coefficients are obtained.

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 4.844 \\ 968.844 \\ 6.733 \\ 19.377 \\ 0.202 \end{bmatrix} \times 10^{-3} \quad (47)$$

**Indices:**

Existence of some indices plays a crucial role in evaluating the efficiency of the sizing and siting on the technical parameters of the network. Hence, some indices are introduced. Introduced indices can illustrate the average of variation of the parameters. In addition, they can identify whether the parameters are in their allowable range or not.

**The Index of ‘The Power Loss’:**

Using this index, the amount of variation of active and reactive power loss which is resulted from installing DG resources can be calculated. This index is defined as follow for the active and reactive power loss (Luis F. Ochoa, 2006).

$$IL_p = \left(1 - \frac{\text{Re}\{Losses_{With DG}\}}{\text{Re}\{Losses_{Without DG}\}}\right) \times 100\% \quad (48)$$

$$IL_q = \left(1 - \frac{\text{Im}\{Losses_{With DG}\}}{\text{Im}\{Losses_{Without DG}\}}\right) \times 100\% \quad (49)$$

where  $IL_p$  and  $IL_q$  are the percentage of the variation of active and reactive power loss, respectively.

**The Index of ‘Voltage Profile Improvement’:**

This index is defined as follow:

$$VP_{II} = \delta \cdot \left(\frac{VP_{With DG}}{VP_{Without DG}} - 1\right) \times 100\% \quad (50)$$

where  $VP_{II}$  stands for voltage profile improvement index. This illustrates the variation of voltage profile after installing DG resources. In addition,  $VP$  is the amount of voltage profile before and after installing these resources, and is calculated as follow:

$$VP = \sum_{i=1}^n V_i \tag{51}$$

where  $V_i$  is the value of each distribution bus in pu.  $\delta$  is defined as an index which identifies whether the voltage profile is in the allowable range or not.

$$\delta = \begin{cases} 1 & (0.95 < V_i < 1.05) \\ 0 & (V_i < 0.95 \text{ or } V_i > 1.05) \end{cases} \quad i = 1, 2, \dots, n \tag{52}$$

As shown in (62), if the voltage profile of even a single bus violates the allowable range, the value of  $\delta$  and consequently the value of VP will be zero. The higher the value of VP, the more the voltage profile is improved.

**The Index of ‘Short-Circuit Level’:**

This index is calculated as follow.

$$ISC = \beta \cdot \left( \frac{I_{SC}^{With\ DG}}{I_{SC}^{Without\ DG}} - 1 \right) \times 100\% \tag{53}$$

Where ISC stands for the index of short-circuit level,  $I_{Without\ DG}^{SC}$  and  $I_{With\ DG}^{SC}$  are the short-circuit level of the network, before and after installing DG resources, respectively.

$$I^{SC} = \sum_{i=1}^n I_i^{SC} \tag{54}$$

Where  $I_i^{SC}$  is the short-circuit current of each distribution bus, before and after installing DG resources. To identify whether the increase in the short-circuit level has exceeded the tolerable amount of the CBs or not,  $\beta$  coefficient is defined as follow.

$$\beta = \begin{cases} 1 & (I_i < I_{Switch,i}) \\ 0 & (I_i > I_{Switch,i}) \end{cases} \quad i = 1, 2, \dots, n \tag{55}$$

If the short-circuit level of all the buses were in tolerable range of the CBs,  $\beta$  would equal to the value of 1. The value of index of the increase in short-circuit level can be calculated using (63), in this situation. If  $\beta$  equals to the value of 0, the short circuit level of at least one bus has exceeded the tolerable range of the CBs. The index of short-circuit would be zero in this situation.

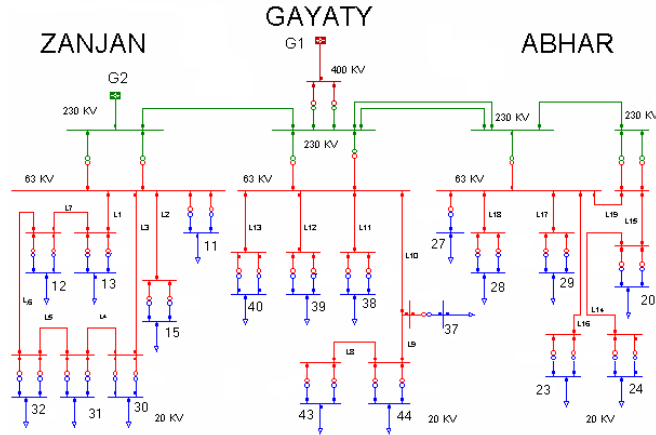
**Proposed Method:**

**Case study:**

To test the proposed method and formulation, actual power network of Zanjan Province in Iran is selected for simulation. Figure 4 depicts the single line diagram of this network. In this network,  $G_1$  bus is considered as Slack bus. The voltage value of this bus is 0.9625 pu.  $G_2$  is considered PV bus and the voltage value is 0.987 pu. The produced power of this bus is 110 MW. The active and reactive loads of this network are 288.55 MW and 93.53 MVar, respectively. Further information of this network is indexed in appendix (Razavi, 2011).

**Software:**

The proposed method and the sample network are simulated in DIgSILENT Power Factory 14.0.523 program. DIgSILENT program is an advanced software package for simultaneous analyzes of power network and control systems. This program is capable of calculating load flow, short-circuit level, active and reactive losses of the network and the parameters of the network. The main feature of this program is the ability of programming, which is called DPL (DIgSILENT Programming Language).



**Fig. 4:** Single line diagram of Zanjan Province network.

**Optimization Technique:**

In this paper, Genetic Algorithm (GA) is used to optimize the objective function. To optimize a problem, using the GA, a population is required to be defined at the first step. This population is formed by binary accidental quantization of chromosomes. In the next step, produced population is applied to the objective function and the fitness of chromosomes is obtained, using (66). Some of the best answers are chosen and new generation is produced by the genetic operators of crossover and mutation. In the first type, two gens, that should be combined, are placed beside each other and are divided from a specified point. Then, the sides that are placed in front of each other are combined together. In the second type, a percent of chromosomes are substituted by another value of their allowable limits, in order to make the optimization, global and not local. To have a global and the fastest answers, both of these genetic operators are used in this paper.

$$Fitness = \frac{1}{f} \tag{56}$$

**The Algorithm:**

To apply the proposed method, the following steps are considered.

1. Losses, short-circuit level and voltage profile, in the absence of DGs, are calculated using load flow program.
2. Proposed numbers, locations and capacities of DGs, which are identified by GA, are applied to the network.
3. If the constraints are satisfied the program will go to step 5 and if not it will go to step 4
4. Answers are rejected and the program returns to step 2.
5. Weighting coefficients are calculated, using equations (48) to (57).
6. The best answers are identified, considering the OF and indices.

**Simulation Results:**

The following capacities, locations and number of DG resources are obtained from applying the proposed algorithm and selected weighting coefficients.

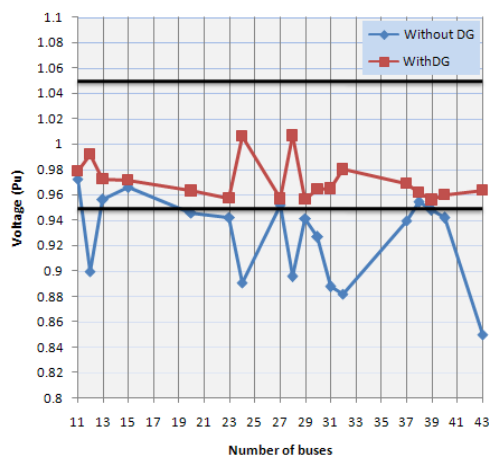
**Table 1:** Obtained number, capacity and location of DG resources.

Location	Capacity (MW)	(MVar) Capacity	$CG_k$ (MVA)
12	15	11.25	18.75
24	15	11.25	18.75
28	10	7.5	12.5
32	10	7.5	12.5
43	15	11.25	18.75
44	10	7.5	12.5

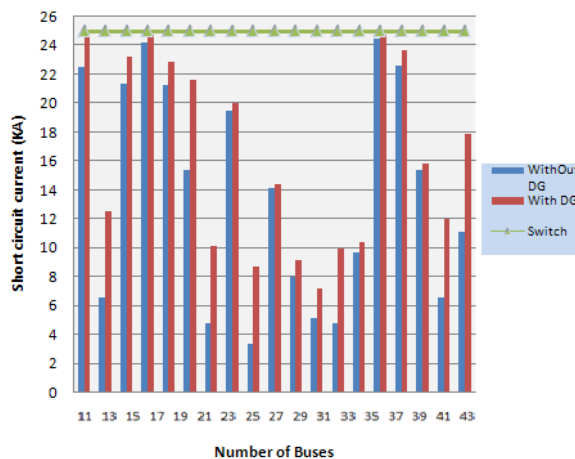
As discussed in the formulation part, the proposed method of this paper is capable of deciding the optimal number, location and capacity of DG resources to obtain the most efficiency of installation. Accordingly, as illustrated in Table 1, to reduce the power loss, diminish the short circuit level and improve the voltage profile simultaneously, 6 DG units with identified locations and capacities are installed. The produced reactive power of these resources is assumed to be 0.8 which is the minimum value, identified by electrical companies.

As depicted in Figure 5, the voltage profile had an inappropriate condition, before installing DG resources and the voltage value of only five buses was in allowable range. The voltage profile improved significantly, after installing DG resources and the voltage of all the buses placed in the allowable range.

Figure 6 depicts the short-circuit level of the buses, before and after installing DG resources. As it is evident, the short-circuit level of the buses increase after installing DG units. The buses, which DG resources are installed on, have the most values of increase. Therefore, these buses should be considered more than the others. As illustrated, all the increases have been in the tolerable range of CBs and consequently, it is not essential to change them.



**Fig. 5:** The voltage profile before and after installing DG resources.



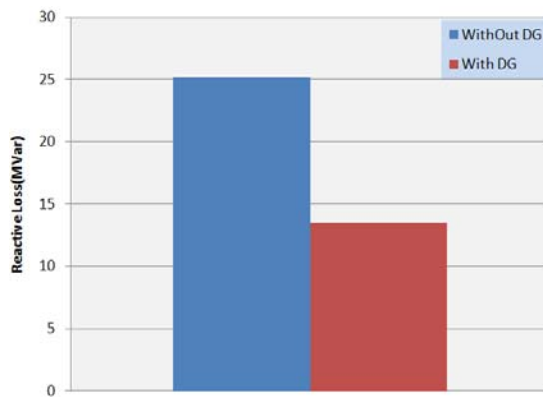
**Fig. 6:** The Short-circuit current of the distributed buses, after and before installing DG resources.

As shown, both of the active and reactive losses have decreased after installing DG resources. The active loss was 10.72 MW and it decreased to 4.11 MW after DG installation. This result also stands for reactive loss. However this factor was not inserted in the OF, but reduction in the current of the lines, which was resulted from installing DG resources, has brought about the reduction of reactive loss. The amount of reactive loss has reduced from 26.62 MVar to 13.17 MVar.

The following figures depict the active and reactive power loss, before and after installing DG resources.



**Fig. 7:** The total active loss of the network, after and before installing DG resources.



**Fig. 8:** The total reactive loss of the network, after and before installing DG resources.

It is apparent from Table 2, that the values of the loading of distribution substations have satisfied the constraint 5. After installing DG resources all the loadings of the distribution substations have reduced. This reduction is as a consequence of supplying a part of the demand load of the network, which is carried out by DGs. For instance, the loading of the substation 12 was out of the permitted range in the absence of DGs and in reduced 89.01 percent, after installing DG resources. Having this amount of decrease, the loading of this substation became 34.05 percent and placed in the permitted range. Therefore, the necessity of replacing the associated transformer was removed.

The efficiency of the performed siting and sizing is evaluated, using the introduced indices. The values of these indices are presented in Table 3. As presented in this table, the voltage profile of distribution network is improved 4.88 percent. Having this improvement, all the voltages of distribution network buses placed in the allowable range. In addition, the active and reactive losses of the network reduced 61.75 and 59.28 percent. Furthermore, the index of short-circuit level increased 20.39 percent.

Since all the buses of distribution network have the capability of installing DG unit, the different values of indices are presented the situation where, one to nineteen DG units are installed. It is apparent from Figure 9 that by increasing the number of DG units, the index of power loss is increased. But, in the reality it is not economical to increase the number of installed DG resources more than an optimal value. On the other hand, this increasing results in the deterioration of the other parameters. Therefore, the other parameters should be considered while deciding the optimal number of DG resources.

The value of VPII becomes zero for less than 6 more than 12 DG units. In these situations, the related constraint is violated. On the other hand the value of ISC becomes zero, installing more than 8 DG units. This also shows that the associated constraint is violated. Consequently, for 6, 7 and 8 DG units, all the constraints are satisfied and the optimal answer should be decided from these values. Considering the value of the OF, it is apparent that installing 6 DG units leads to the most appropriate value of the OF. These values are presented in Table 4. Hence, the optimal number of DG units which provide the maximum technical advantages is acquired, using the proposed method.

**Table 2:** Loading of MV substations, before and after installing DG resources.

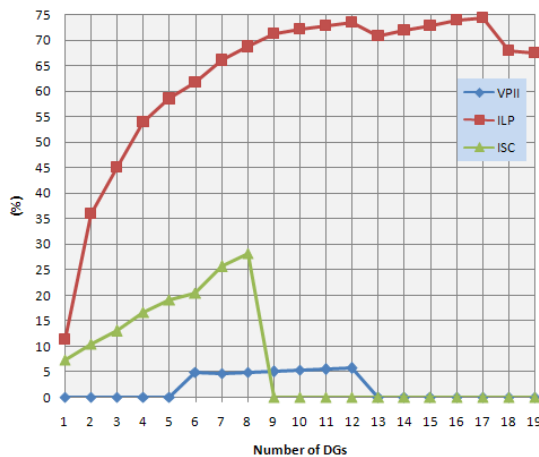
Number of bus	Loading Before(%)	Loading After(%)
11	35.24	34.7
12	123.06	34.05
13	28.93	28.45
15	42.07	41.82
20	37.9	37.21
23	59.38	59.06
24	89.15	60.97
27	54.7	54.41
28	62.65	38.95
29	26.17	26.03
30	23.91	22.98
31	24.96	22.97
32	25.13	23.92
37	2.65	2.57
38	35.24	34.94
39	42.34	41.98
40	72.57	71.94
43	42.35	14.98
44	48.41	21.47

**Table 3:** The indices of sizing and siting.

$IL_p$	$IL_q$	VPII	ISC
61.75	59.28	4.88	20.39

**Table 4:** The values of the OF.

Number of DG	OF value
6	0.022910
7	0.023700
8	0.025735



**Fig. 9:** Different value of indices for different number of DGs.

**Conclusion:**

In this paper, an innovative Objective Function is proposed in order to identify the optimal location, capacity and number of required DG resources. Using the proposed algorithm of this paper, the optimal number of DG units, in order to obtain the maximum technical advantages of installing these resources. The technical considered objectives of this paper is according to the Iran electrical companies requirements and includes, reduction of power losses, improvement of voltage profile, and reduction of short-circuit level. The proposed method identifies different constraints of the network and minimizes the essential changes that should be performed after installing DG units.

The proposed method is applied to the actual power network of Zanjan Province in Iran and the simulation results shows that using the proposed method of this paper, an optimal number, place and capacity of DG resources are acquired. In addition, some indices are presented and applied to evaluate the effectiveness of the method. Calculating this index for different number of DG units, it is proven that the most appropriate answer for the OF is obtained having the calculated number this paper for DG units. The proposed algorithm of this paper is applicable to any siting and sizing procedure and in any network.



**Appendix:**

**Table 5:** Data for the 63KV lines.

Line number	Line length (km)	Active loss (MW)
L1	10.6	0.524
L2	4.5	0.123
L3	66.3	0.407
L4	84.15	0.323
L5	19.9	0.002
L6	51.5	0.331
L7	46.5	0.72
L8	40	1.26
L9	14	1.028
L10	11	0.8
L11	6	0.117
L12	8.3	0.239
L13	10.3	0.233
L14	45.3	0.379
L15	9	0.537
L16	11.3	0.52
L17	23.8	0.287
L18	116.6	0.513
L19	28.5	0.317

**Table 6:** Data for the voltage, active power and reactive power of distribution buses before installing DG resources.

Bus number	Bus voltage (pu)	Active power (MW)	Reactive power (MVAR)
11	0.9726	21	7
12	0.8993	15.4	6.2
13	0.9561	15.4	6.2
15	0.9659	22.5	9.4
20	0.9456	20.5	6.5
23	0.9423	33	6.2
24	0.8905	11.6	2.7
27	0.9519	12	10
28	0.8955	8.2	1.9
29	0.9411	14.2	4.1
30	0.9268	6.5	1.4
31	0.8876	6.5	1.4
32	0.8818	6.5	1.4
37	0.9393	0.35	0.13
38	0.9541	19.2	6.2
39	0.9481	22.5	8.6
40	0.9418	19.2	7.2
43	0.8495	21	5
44	0.9055	13	2

**Table 7:** Short circuit level before installing DG resources and short circuit level of CBs.

Bus number	Short circuit level before installing DG resources (kA)	Short circuit level of the CB (kA)
11	22.5	25
12	6.52	25
13	21.31	25
15	24.2	25
20	21.22	25
23	15.4	25
24	4.8	25
27	19.49	25
28	3.36	25
29	14.12	25
30	8.01	25
31	5.11	25
32	4.76	25
37	9.62	25
38	24.43	25
39	22.62	25
40	15.34	25
43	6.54	25
44	11.09	25

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